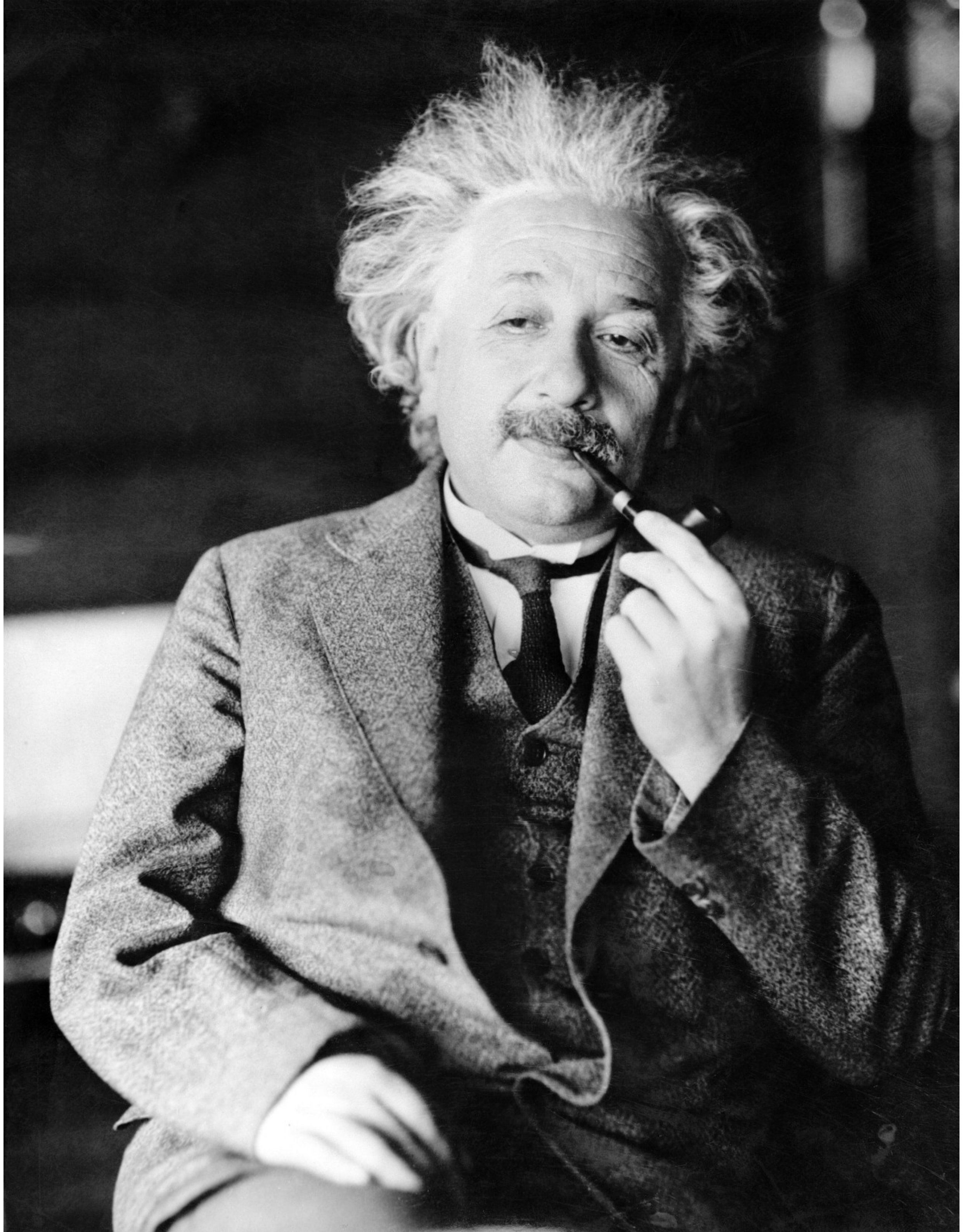
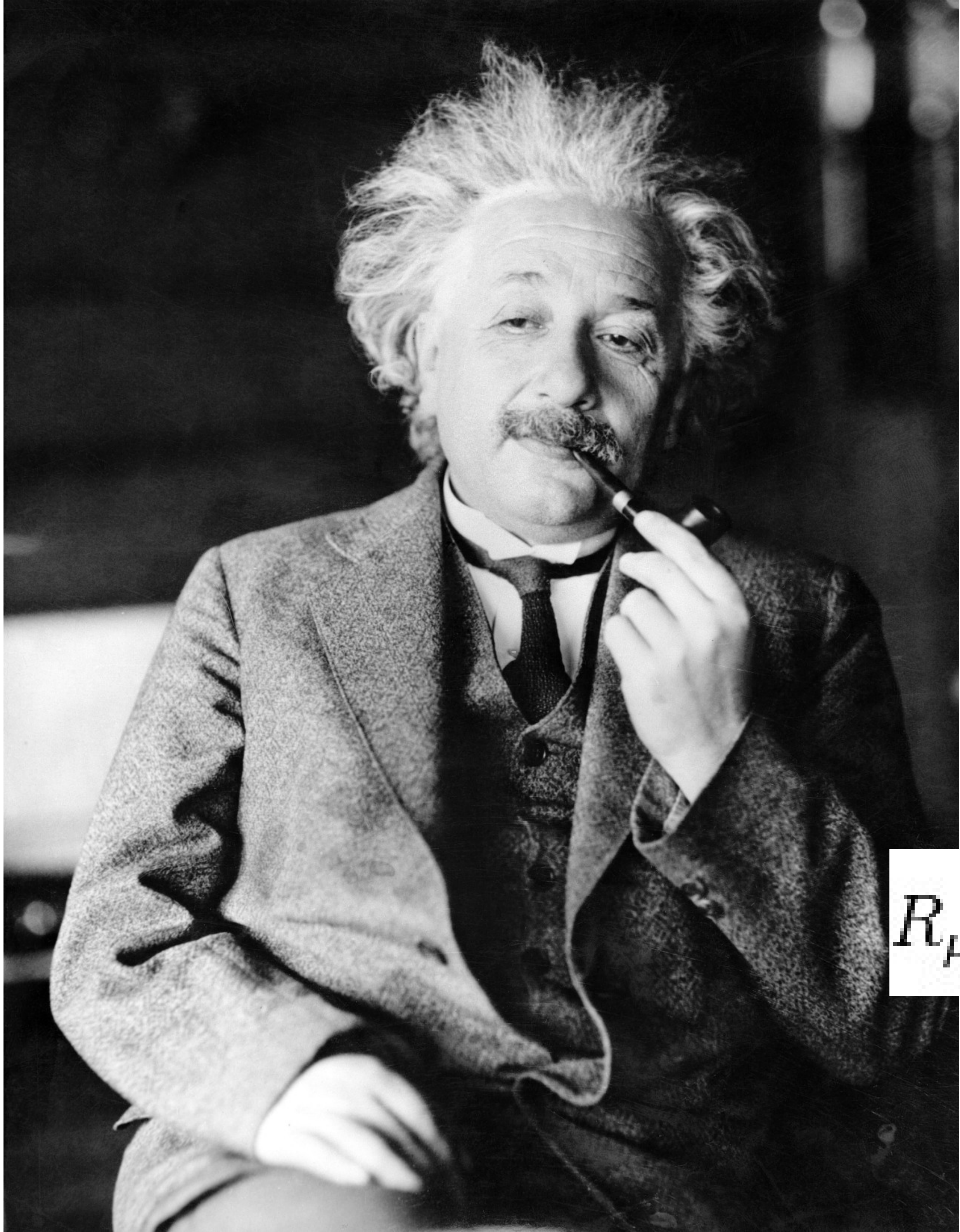
KCL

COSMOLOGY HACKATHON

JUNE 2026



What is Einstein’s
most important equation?

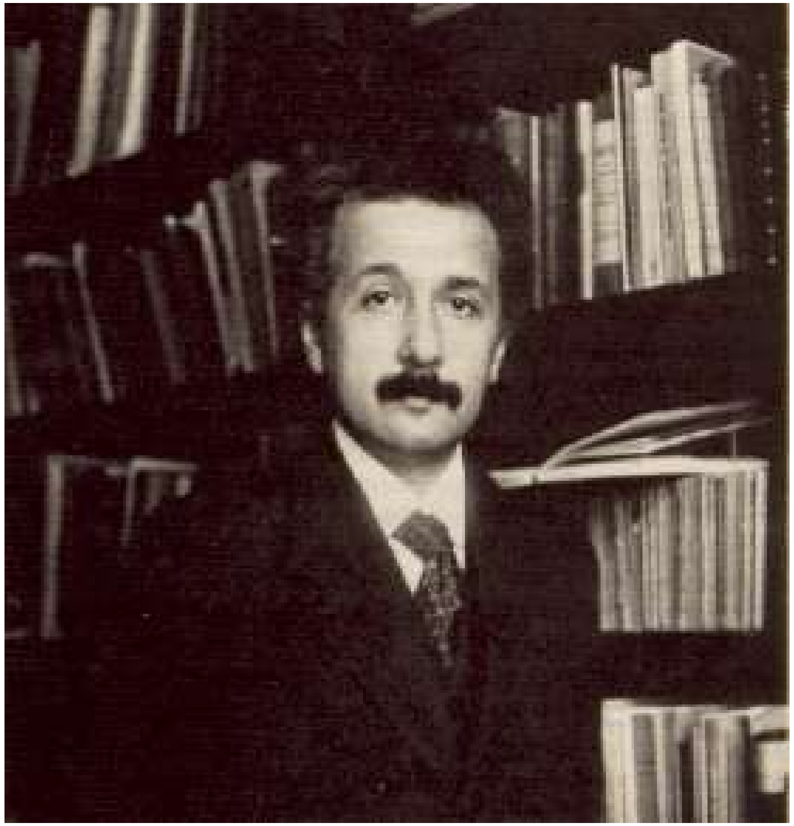


What is Einstein's
most important equation?

$E=mc^2$

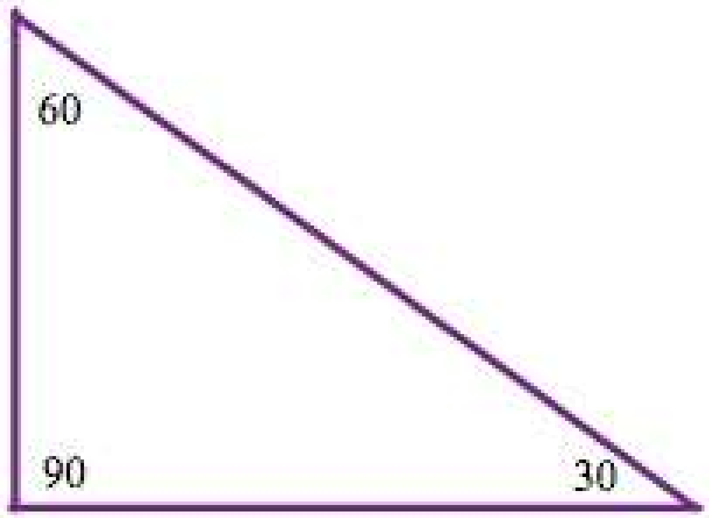
NO!

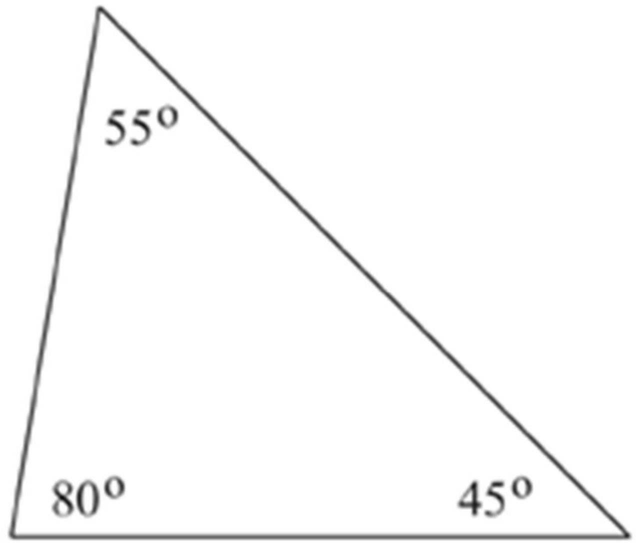
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

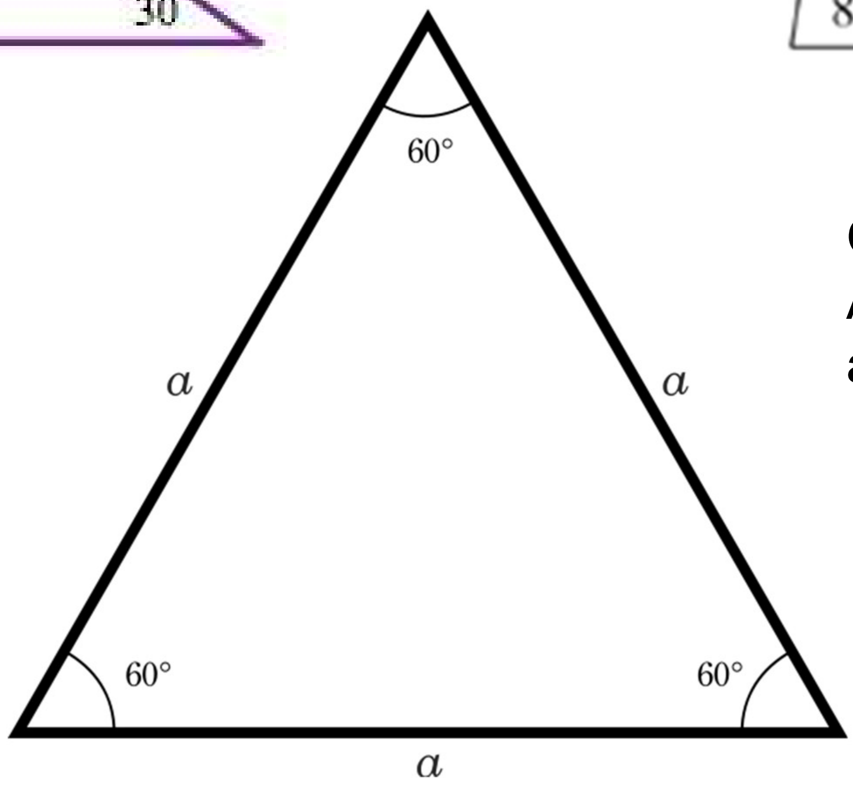


Einstein believes that Gravity is not a force, rather particles appear to experience force as they follow ‘straight lines’ through a curved space.

Curvature



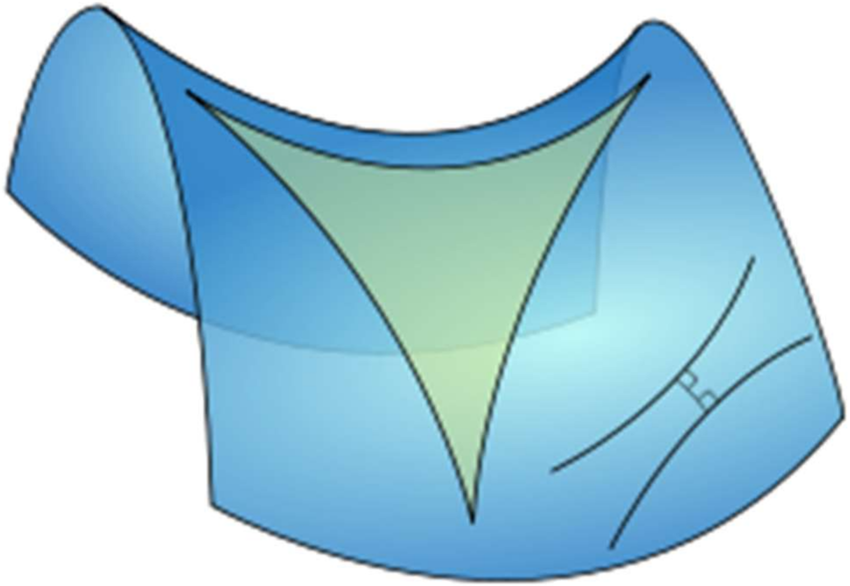




On a flat 2D surface, the Angles of a triangle always add up to 180^o

Curvature

However on a curved surface, this is not true.

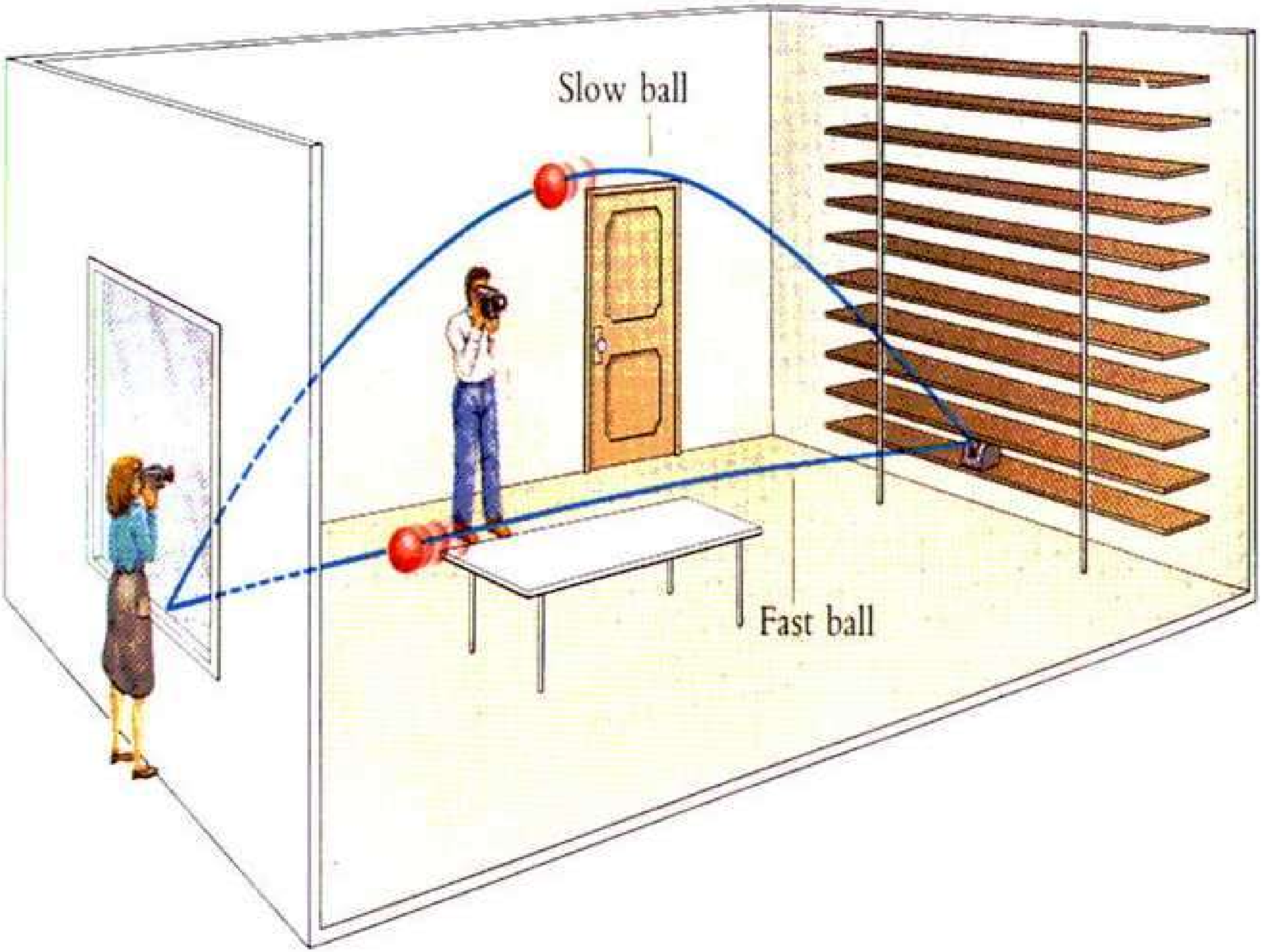


Negatively curved space
sum of angles < 180°

Positively curved space
sum of angles > 180°

Space-Time curvature vs. Space curvature

Actually, we are more familiar with space-time curvature than space curvature!



The Metric

The function which tells us the deviation from pythgoras's law, and other aspects of the curvature

$$ds^2 = f_{xx}(x,y)dx^2 + f_{yy}(x,y)dy^2$$

Can also be written:-

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu$$

Where $g_{\mu\nu}$ is many functions, 4 x 4

$$g_{\mu\nu} = \begin{pmatrix} -\left(1-\frac{r_s}{r}\right) & 0 & 0 & 0 \\ 0 & \frac{1}{\left(1-\frac{r_s}{r}\right)} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix}$$

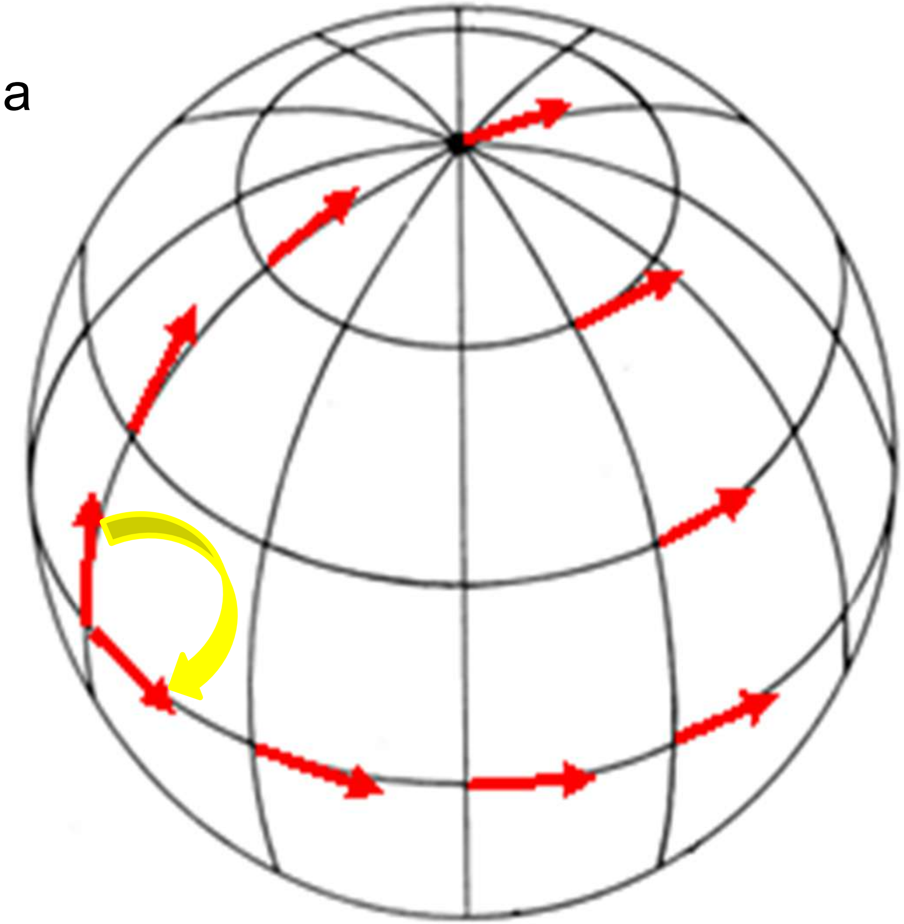
So that

$$ds^2 = -\left(1-\frac{r_s}{r}\right)dt^2 + \frac{dr^2}{\left(1-\frac{r_s}{r}\right)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

Note, non-trivial metric function in front of time coordinate, as well as space!

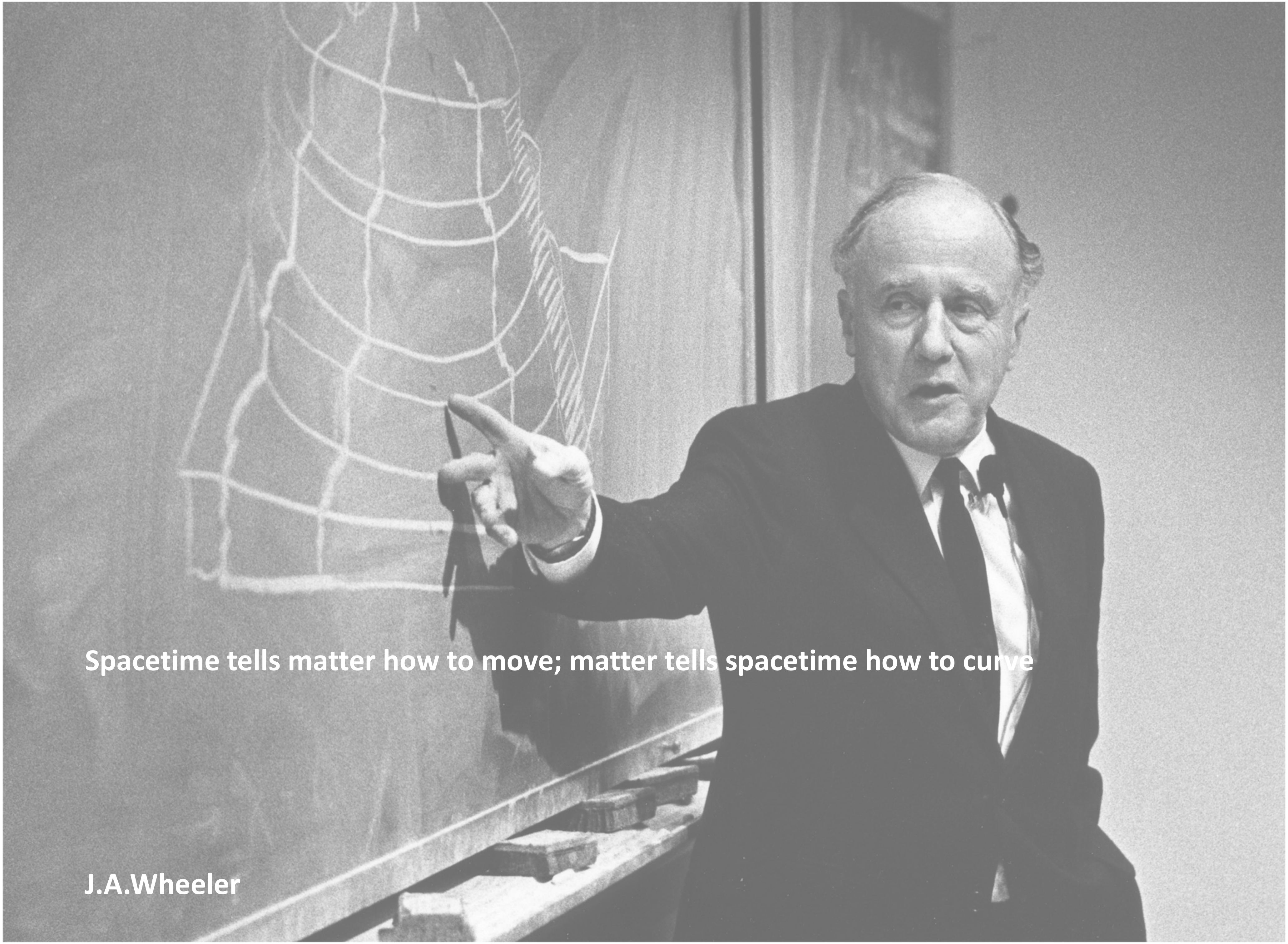
Measuring Curvature

Take a vector round a closed loop and see how it changes



$$Rv = \frac{\nabla}{\partial x}\frac{\nabla}{\partial y}v - \frac{\nabla}{\partial y}\frac{\nabla}{\partial x}v$$

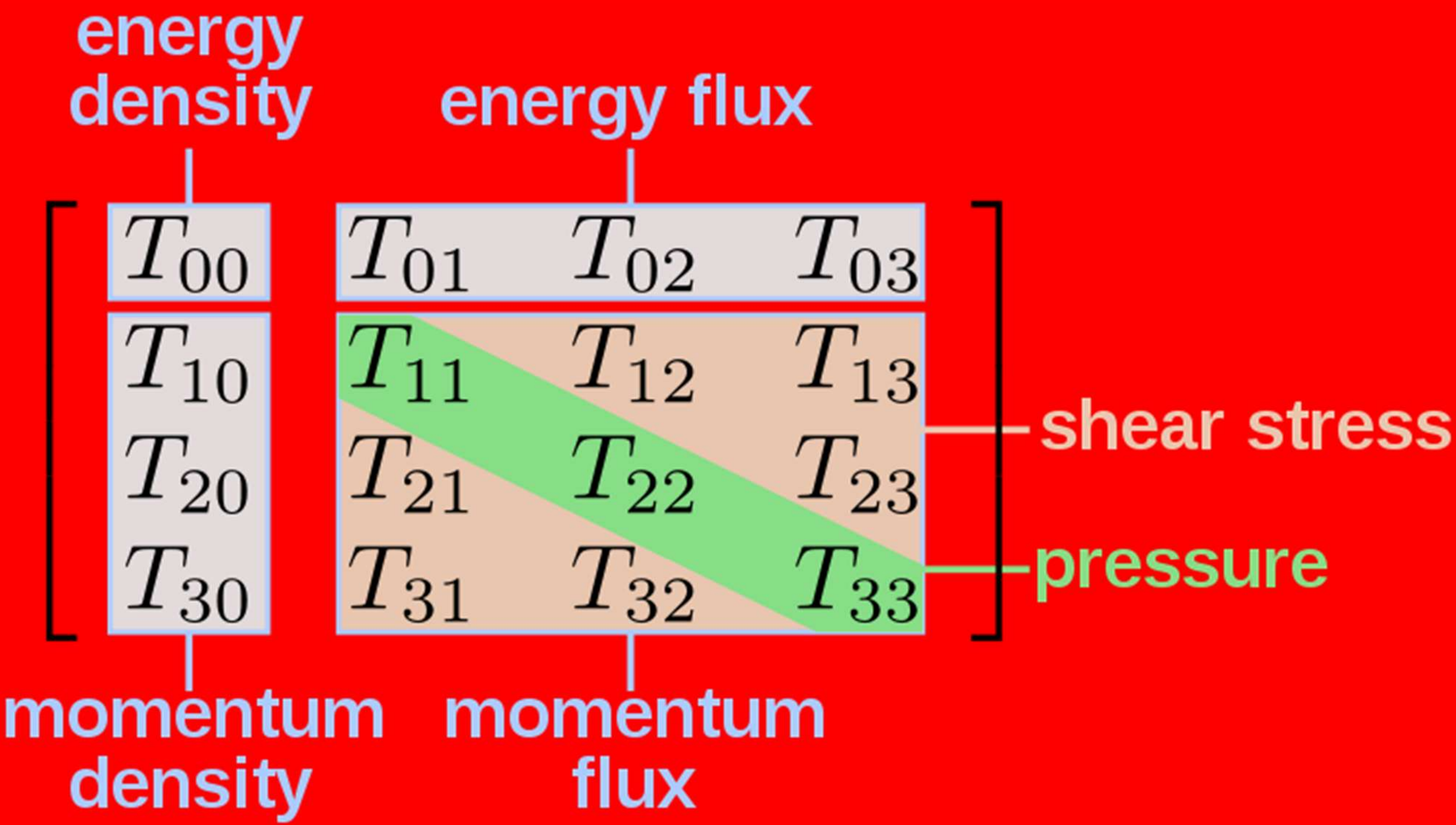
curvature tensor


Spacetime tells matter how to move; matter tells spacetime how to curve
J.A.Wheeler

Stress Energy Tensor

So now we can measure space and space-time curvature. Go back to original idea. Curvature is meant to explain paths followed by particles in gravitational fields. Curvature must be therefore linked to Mass.

Special relativity has told us that Mass and Energy is equivalent, so curvature must be linked to all forms of energy, including mass, momentum, pressure,



Einstein Field Equation(s) (schematic)

$$R = \frac{8\pi G}{c^4} T$$

CURVATURE

ENERGY

We figure out the value of the constant C by looking at the weak field limit (e.g. Near the earth) and comparing it to Newton's Law of Gravity.

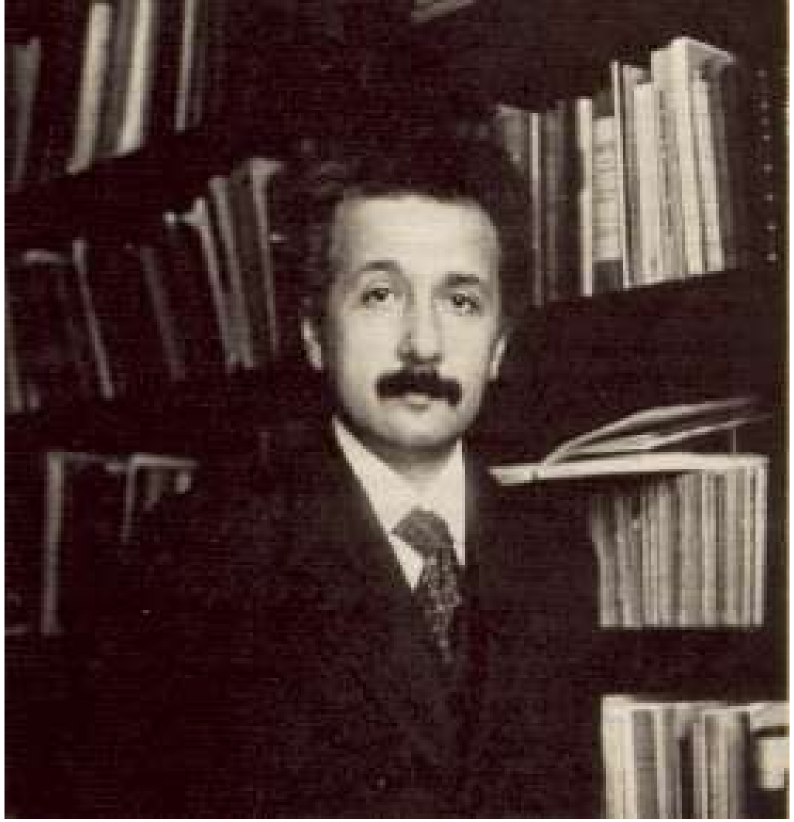
Einstein Field Equation(s)

There are 16!!!

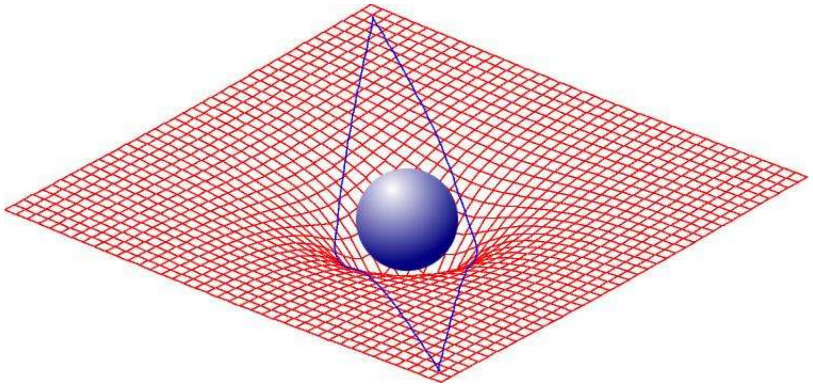
$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

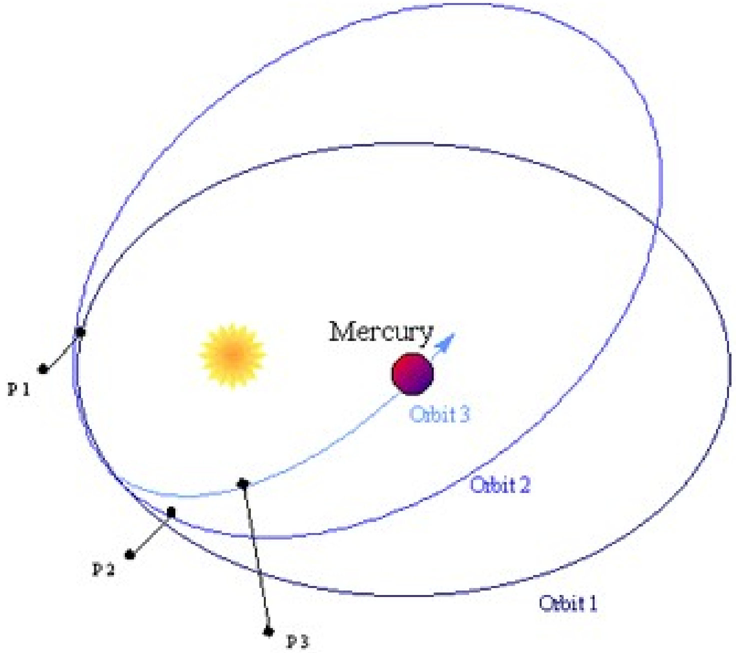
CURVATURE **ENERGY**

We figure out the value of the constant C by looking at the weak field limit (e.g. Near the earth) and comparing it to Newton's Law of Gravity.



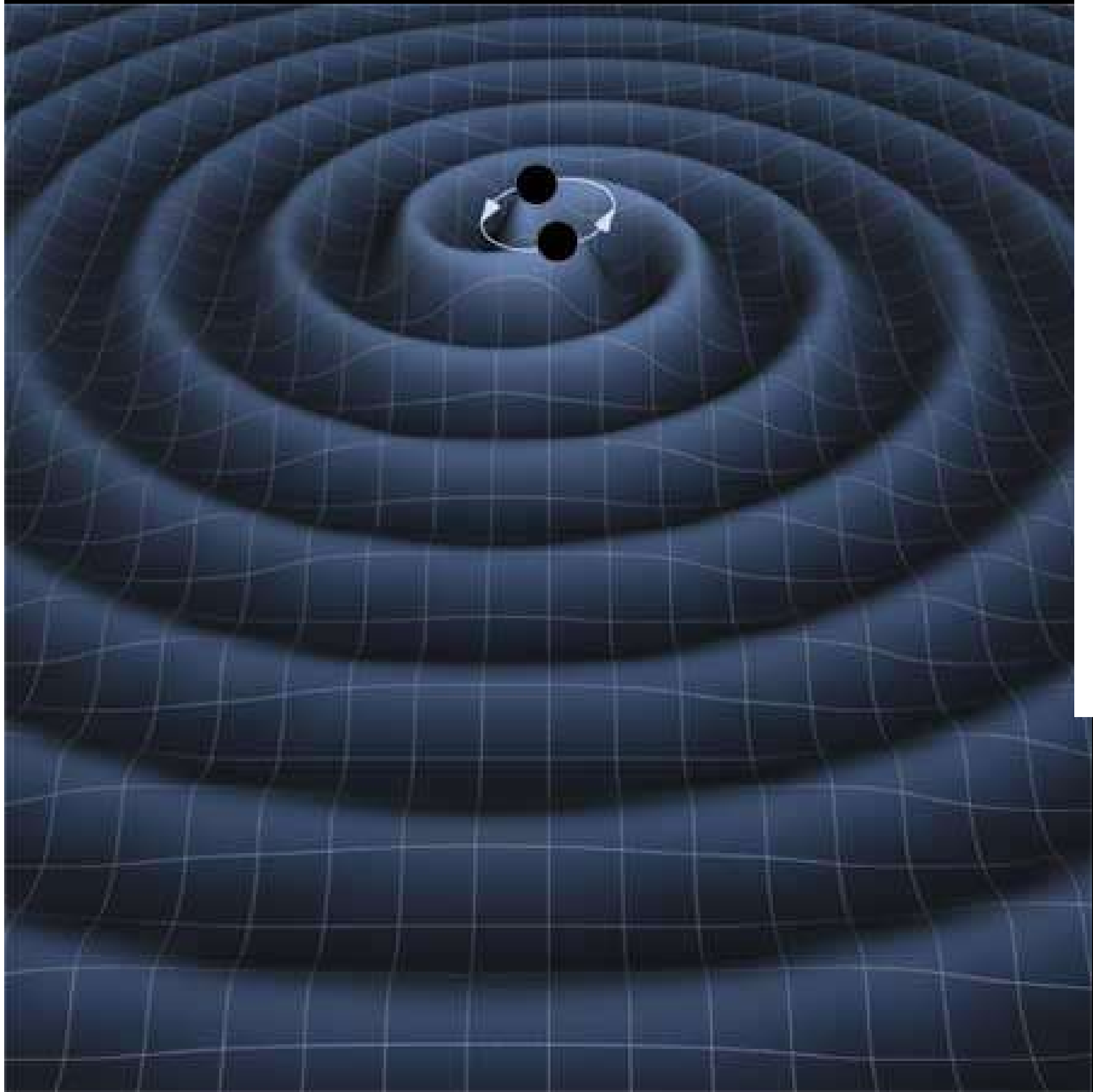
General relativity is extremely succesful

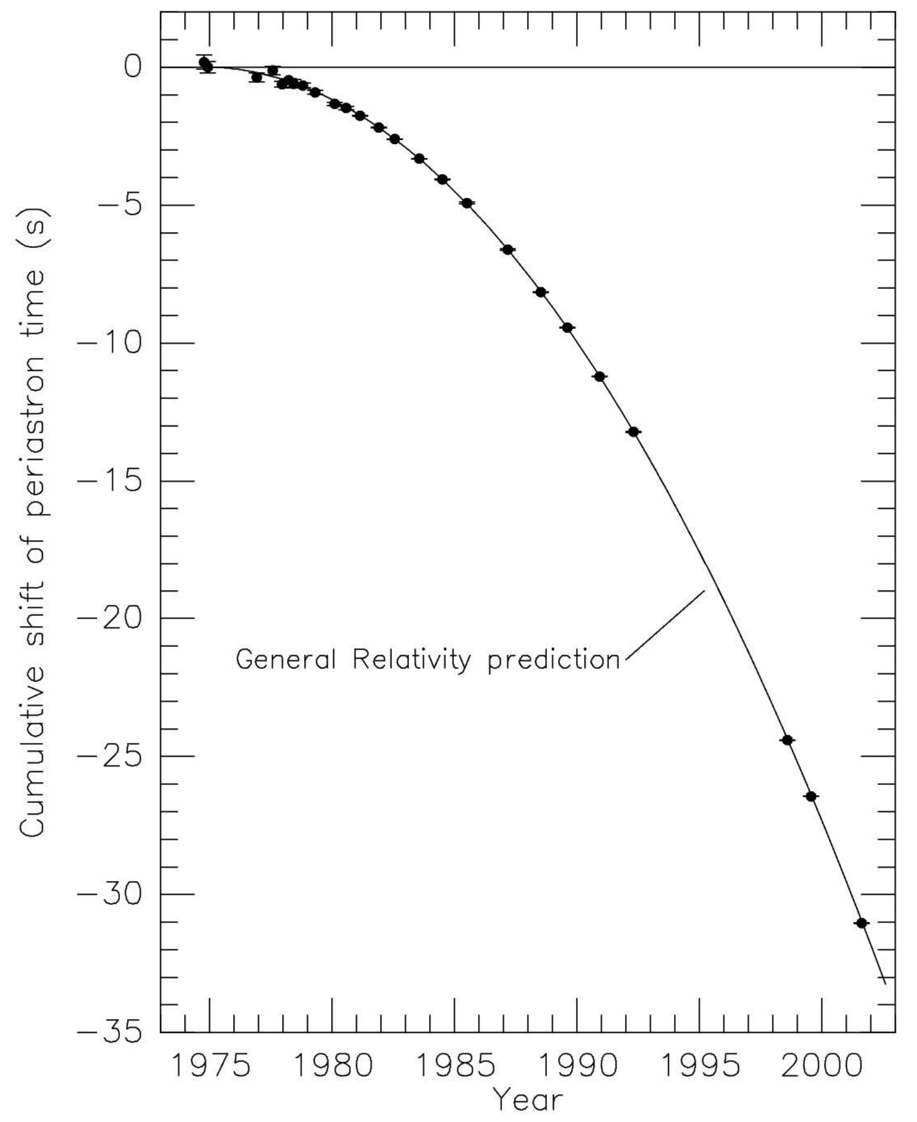




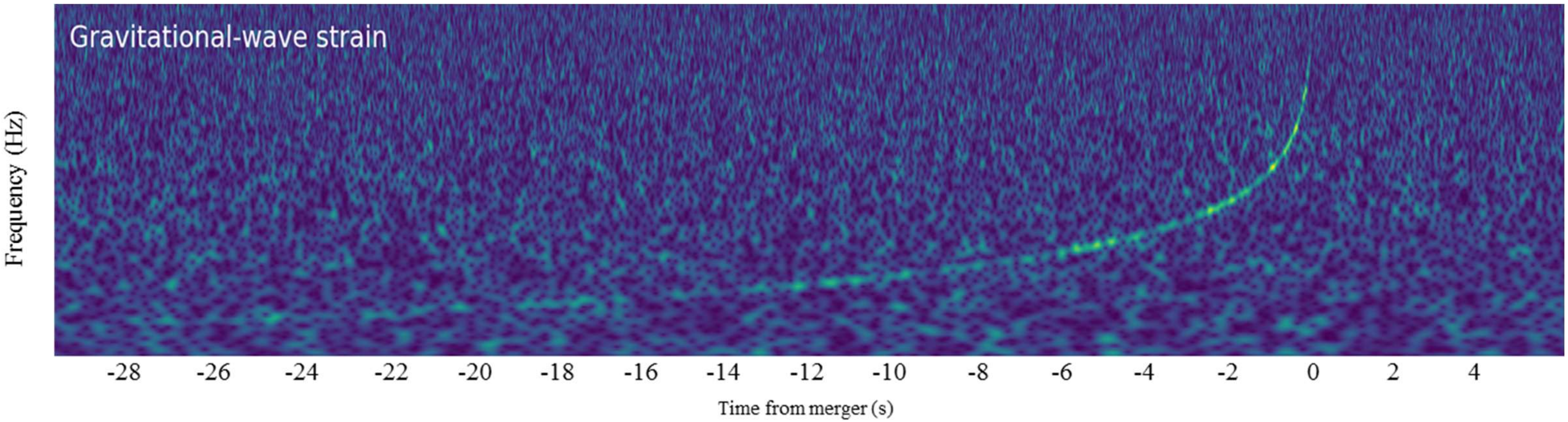
Predicts bending of light round sun… and the orbit of mercury

GR predicts gravitational waves, which should be emitted by binary star systems, resulting in them losing energy and spiralling inwards....





...and they do



Omega Scan of LIGO's First Neutron Star Merger Detection

Friedman
(1888-1925)

Solving GR for whole Universe we get Friedman equations:-

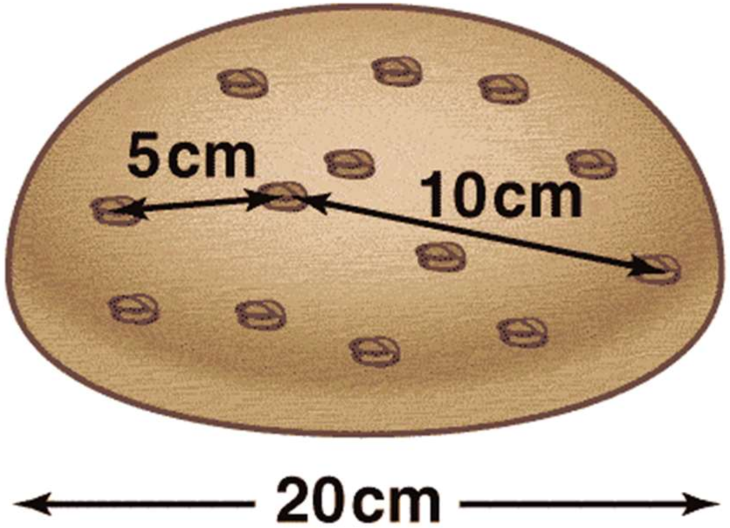
$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3} - \frac{\kappa}{a^2},$$

Friedman equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) .$$

Raychaudhuri equation

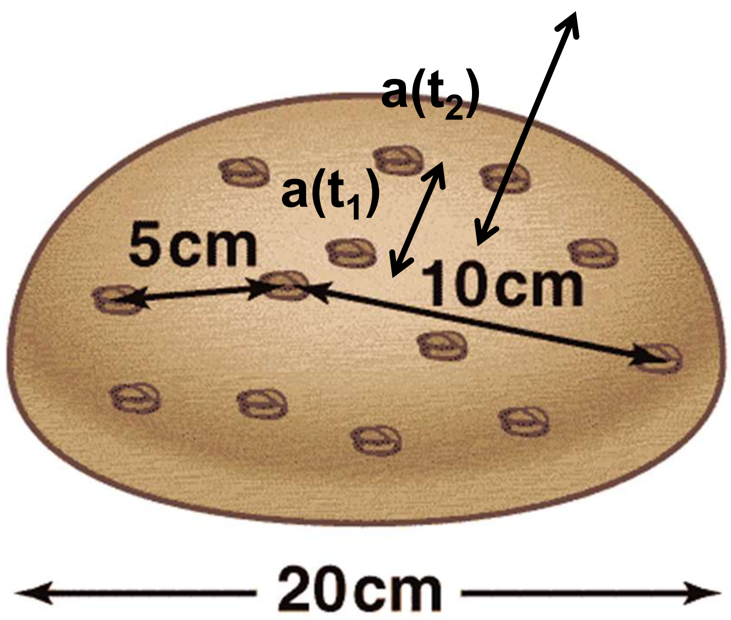
Friedman
(1888-1925)



Imagine that the expanding Universe is a loaf of raisin bread. When baked in the oven, the bread expands, but the raisins do not. The bread represents the space in the Universe, and the raisins represent galaxies and other astronomical objects . While the bread itself undergoes a large change in structure, the raisins themselves stay the same.

MAP990404

What is the scale factor a(t)?



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MAP990404

a(t) is just some arbitrary length, e.g. Distance between two distant galaxies. We keep track of how it changes over time.

What is $\dot{a}$? ("a-dot")

$$\dot{a} = \frac{da}{dt}$$ is the rate of change of the scale factor over time.

If $\dot{a}$ is bigger than zero, things are getting further away from each other so the Universe is expanding. If $\dot{a}$ were negative, the universe would be contracting.

What is $\ddot{a}$? ("a-double-dot")

$$\ddot{a} = \frac{d(\dot{a})}{dt} = \frac{d}{dt}\left(\frac{da}{dt}\right) = \frac{d^2a}{dt^2}$$

This is the rate-of-change-of-the-rate-of-change-of-a! If $\ddot{a}$ is bigger than zero the expansion of the Universe is accelerating (getting faster) and if $\ddot{a}$ is less then zero, the expansion of the Universe is decelerating (getting slower).

What is the Hubble parameter H(t)?

$\dot{a}$ can be bigger or smaller depending upon what a we are talking about. We need to do something about this.

$\frac{\dot{a}}{a}$ solves the problem nicely. We therefore define the Hubble parameter:-

$$H = \frac{\dot{a}}{a}$$

Because the definition of the scale factor is so arbitrary, we try to only use equations that take the ratio of the scale factor at different times.

What is the Hubble constant H_0?

H_0 is the value of H **today**. The velocity of recession of an object is $v = H_0 d$

Note, this only works for relatively nearby objects where we can neglect change in expansion rate over time. Units of H are $\mathrm{kms^{-1}Mpc^{-1}}$

Friedman equation:-

Friedman
(1888-1925)

To work out expansion rate

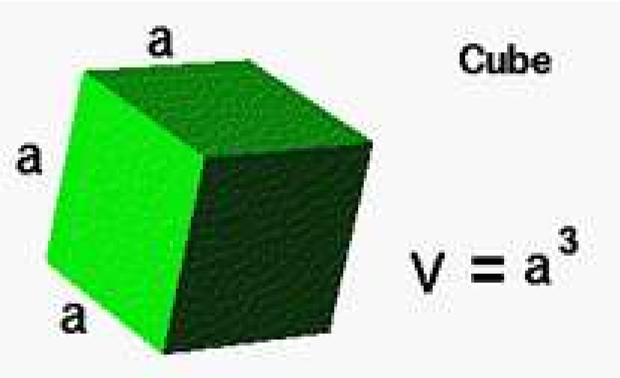
We need to know the density of stuff

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2}$$

and the spatial curvature

However, density is mass/volume and the volume is changing

$$\rho = \frac{M}{V}$$

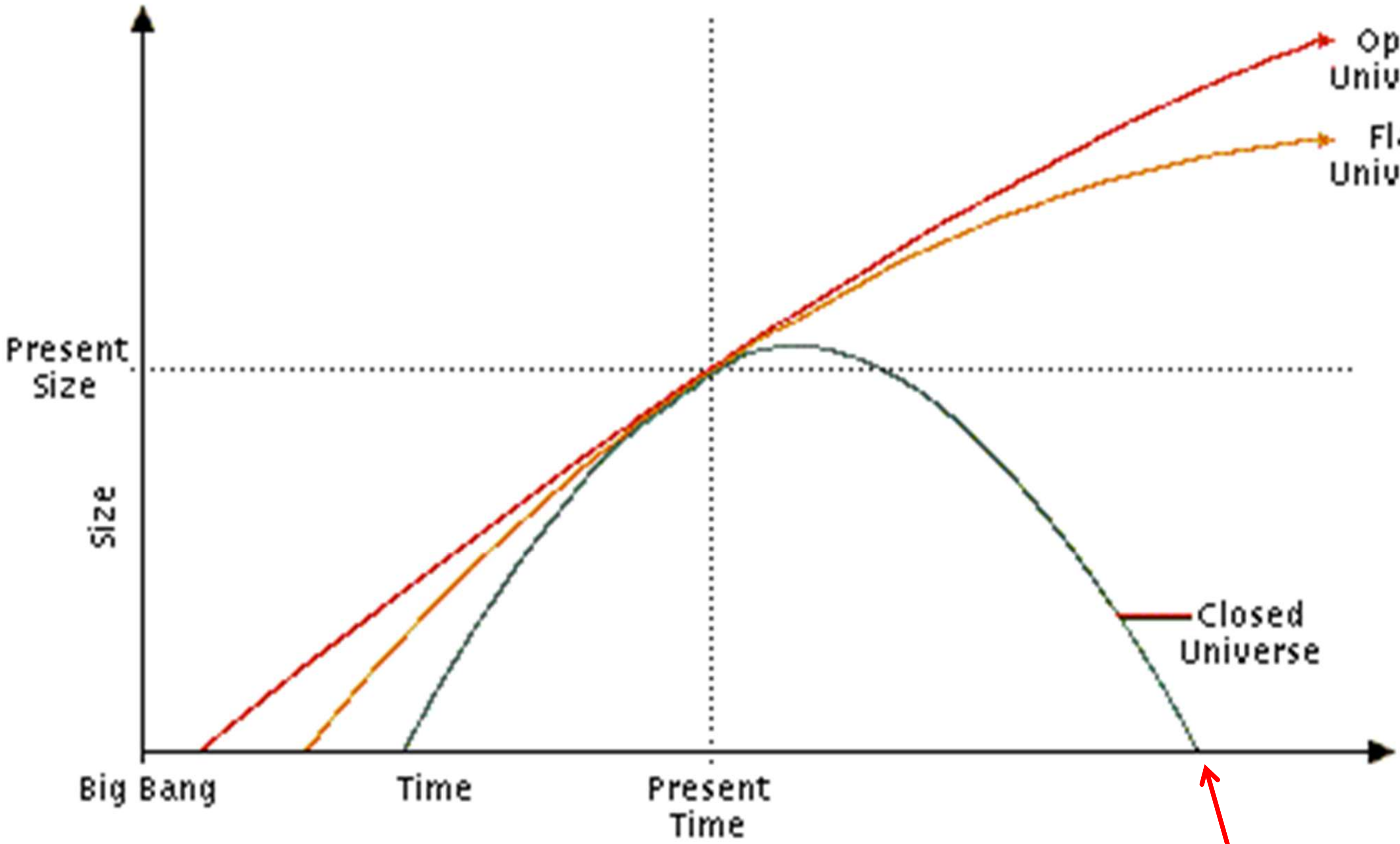


So as the Universe expands, the density of ***matter*** gets diluted as the inverse of the cube of the scale factor.

$$\rho(t) = \rho_0 \frac{a_0^3}{a(t)^3}$$

Fates of Differently Curved Universes (containing normal matter)

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2}$$



Only if k=+1 will expansion stop, turn around and lead to **big crunch**

Historical Interlude

1. Einstein develops General Relativity. $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$

2. Friedman, Robertson, Walker and Lemaitre show that this implies Universe is expanding.

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2}$$

3. Everyone realises this means the Universe either begins or ends.

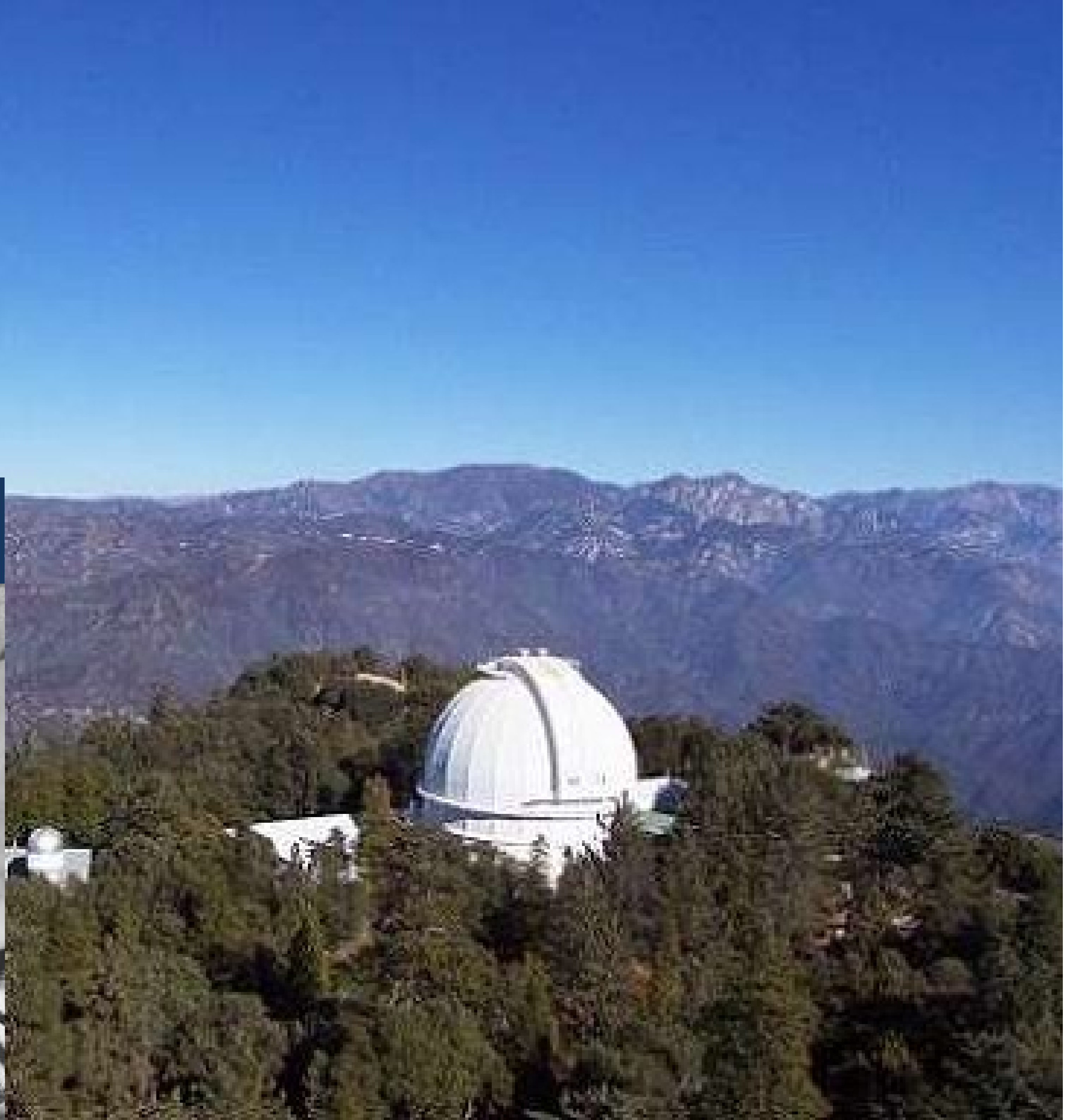
4. Einstein freaks out.

5. Einstein adds an additional term to his equations which only changes cosmology

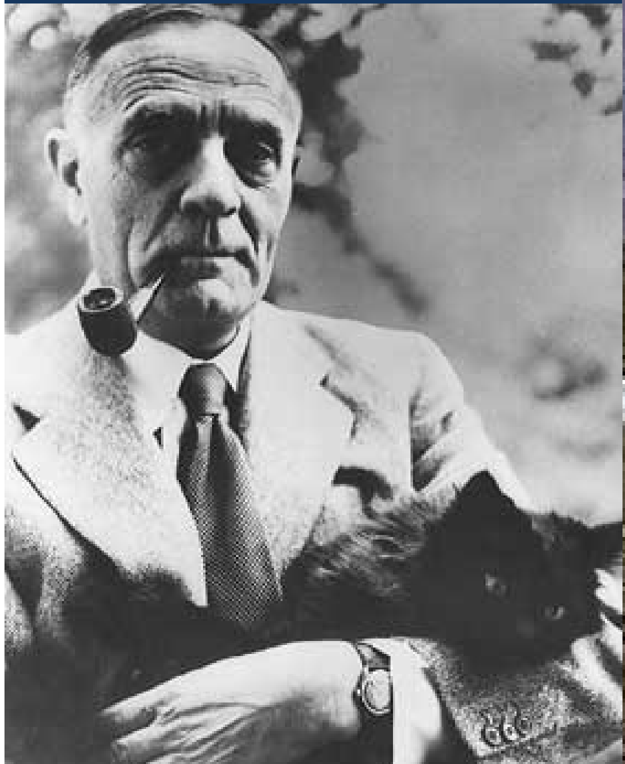
$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

6. By saying k=+1 and Λ>0 he can just about get the Universe to sit still, but it is contrived.

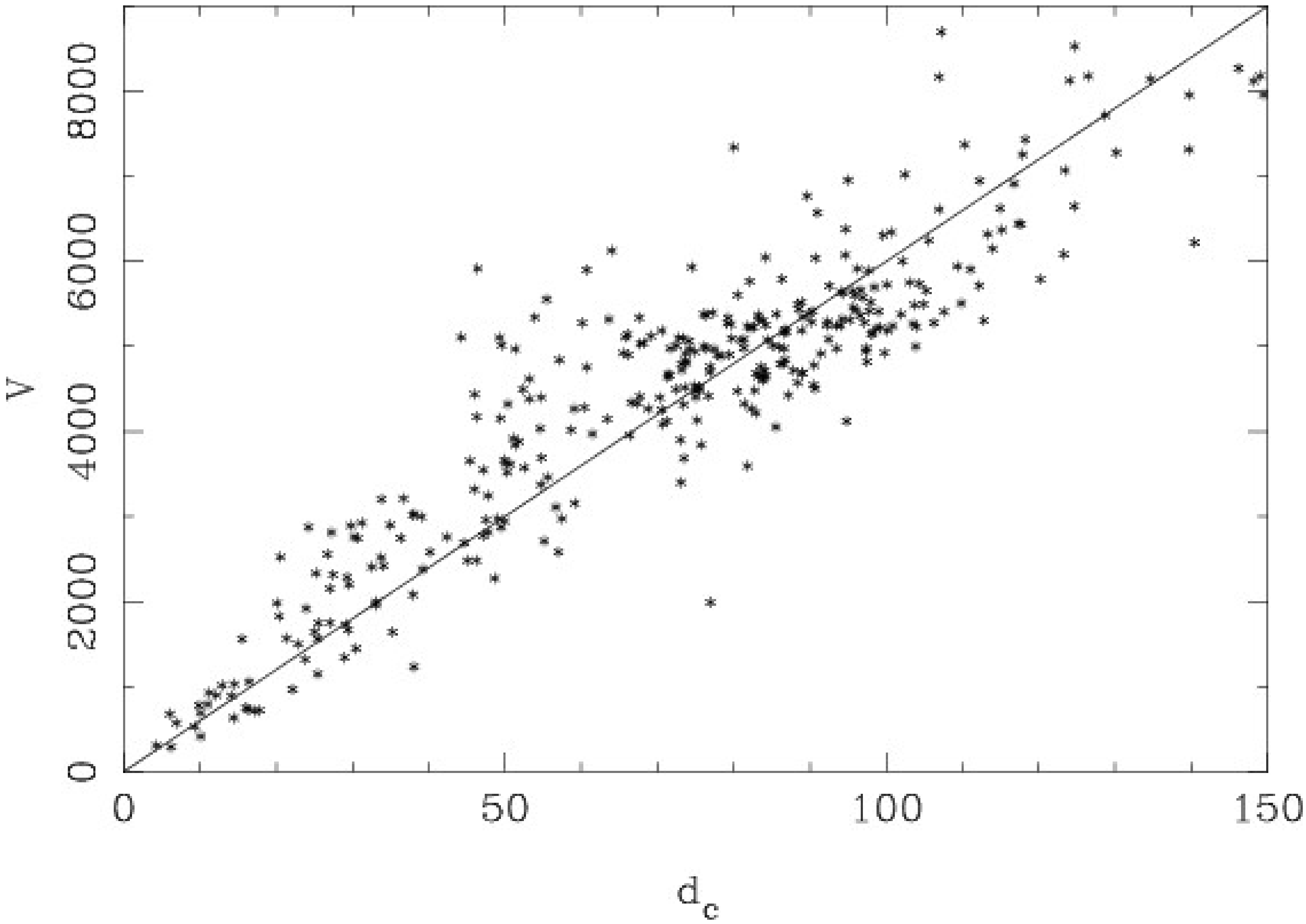
7. Hubble observes with a telescope that the Universe really is expanding.

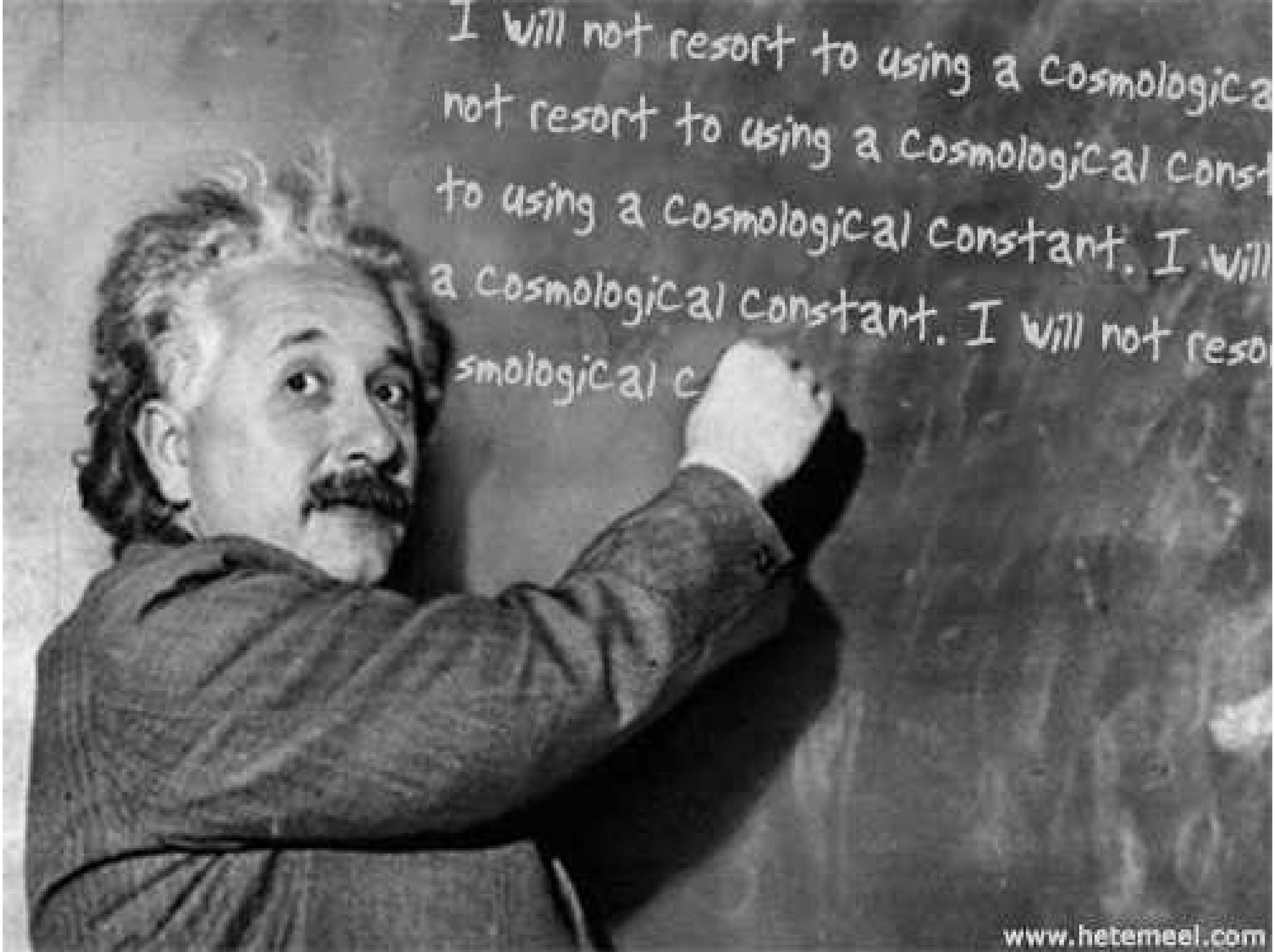


Edwin Hubble
(1889-1953)

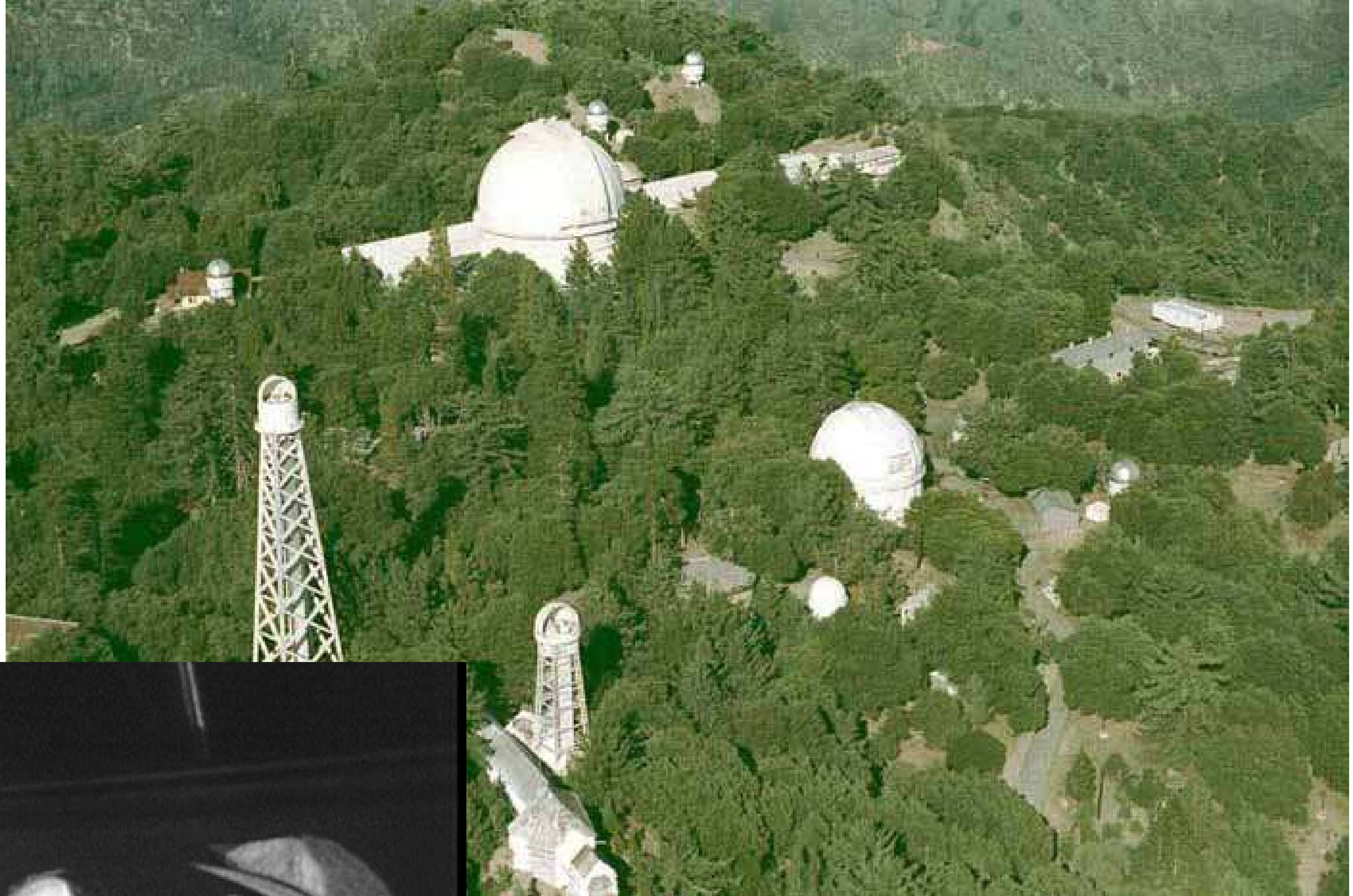


Galaxies actually are moving away from each other (and us)



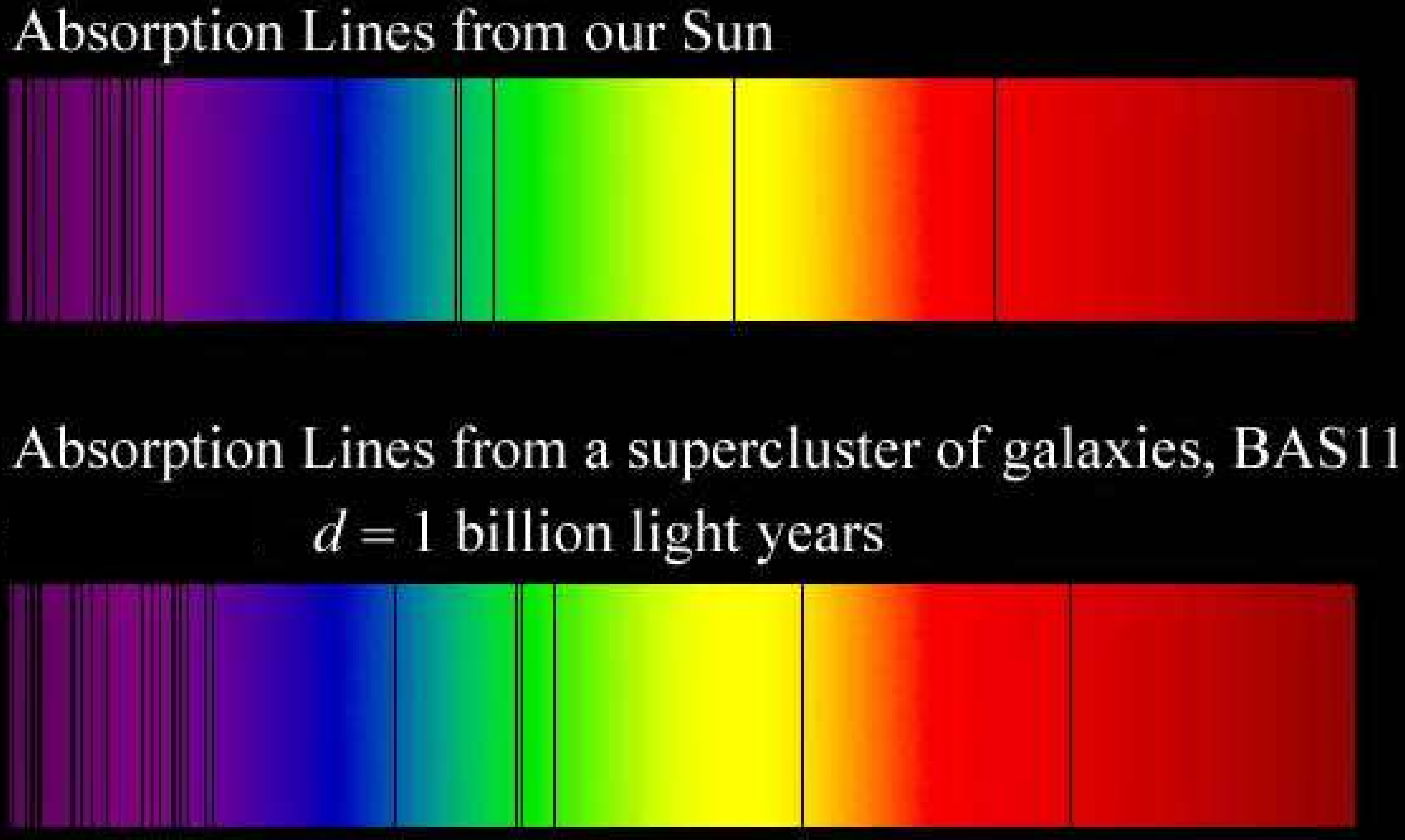

I will not resort to using a cosmologica
not resort to using a cosmological cons
to using a cosmological constant. I will
a cosmological constant. I will not reso
smological c
www.hetemeel.com

Mount Wilson Observatory, Overlooking Los Angeles

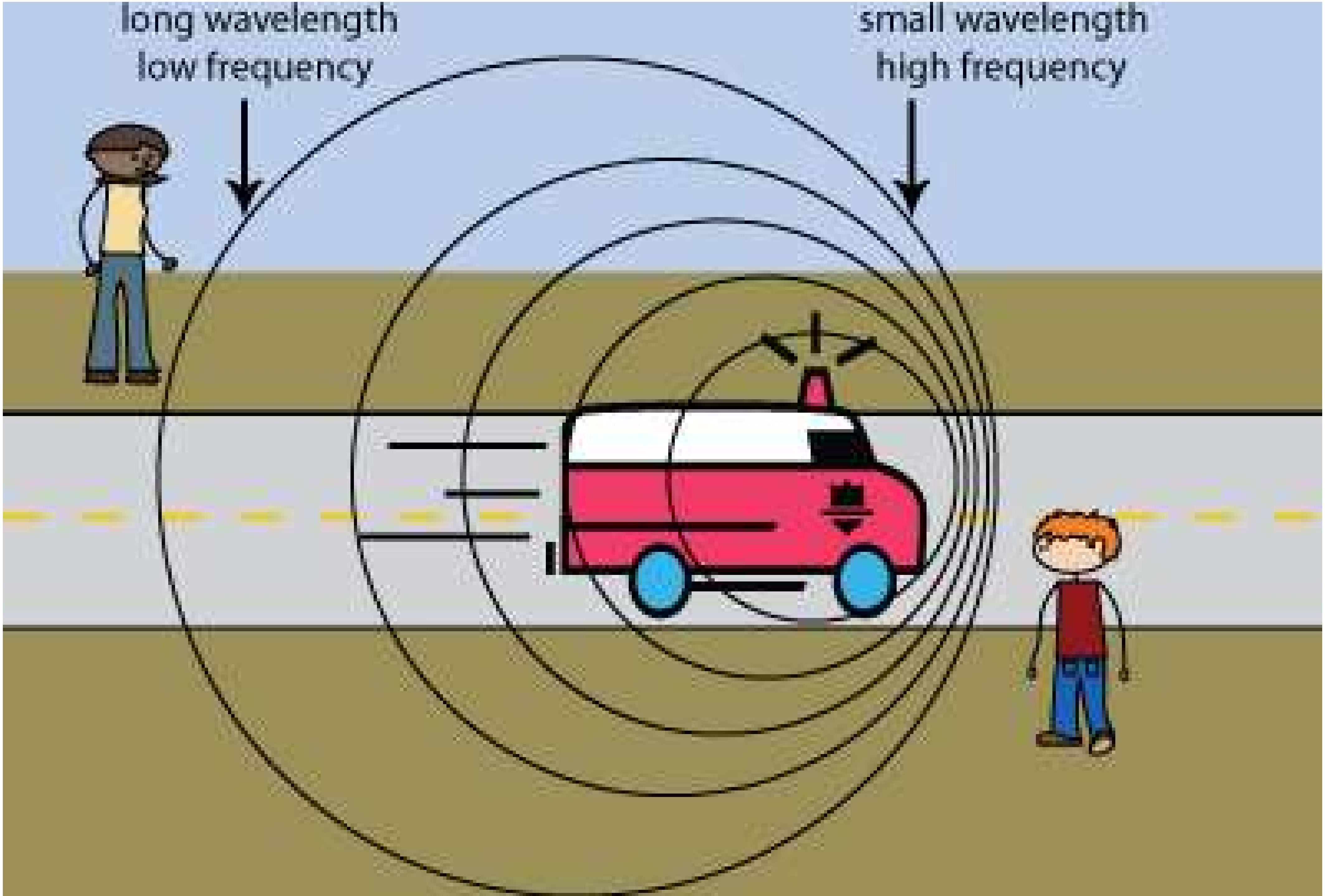


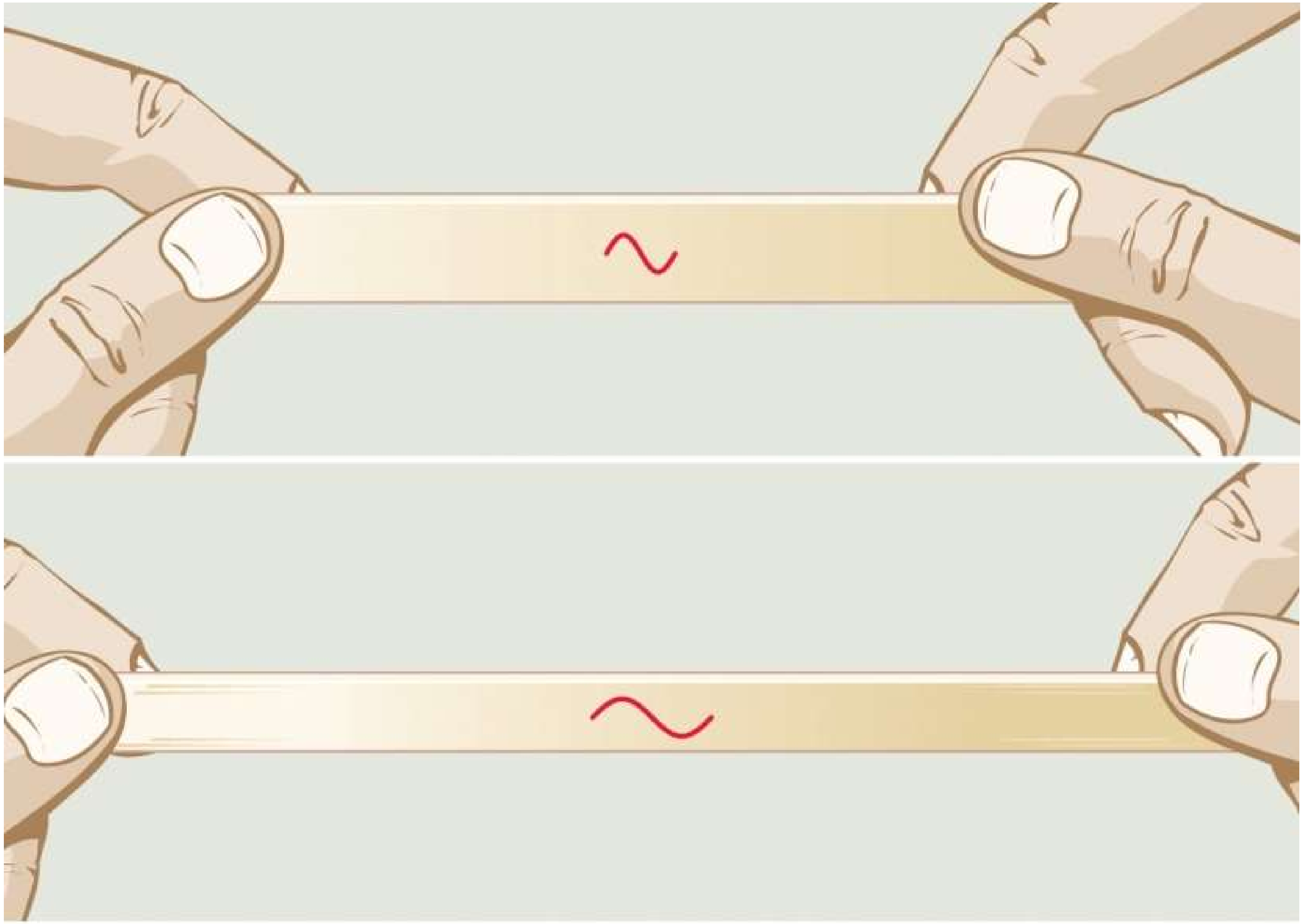
How did Hubble find out the Universe was expanding?

What is Redshift?



Strictly speaking, Cosmological Redshift is different from Doppler Redshift, shown here





Redshift

Well, first of all, we call it z! Let's say we are seeing light from a galaxy at a redshift z.

Scale factor today.

$$z = \frac{a_0}{a} - 1$$

Scale factor when light was emitted.

z=1, scale factor of the Universe when the photon was emitted was 1/2 size it is today.
z=2, scale factor of the Universe when the photon was emitted was 1/3 size it is today.
z=3, scale factor of the Universe when the photon was emiited was 1/4 size it is today.

We tell the redshift by the wavelength λ of the light observed. We need to know what λ it had when it was emitted, but often we do (atomic transitions).

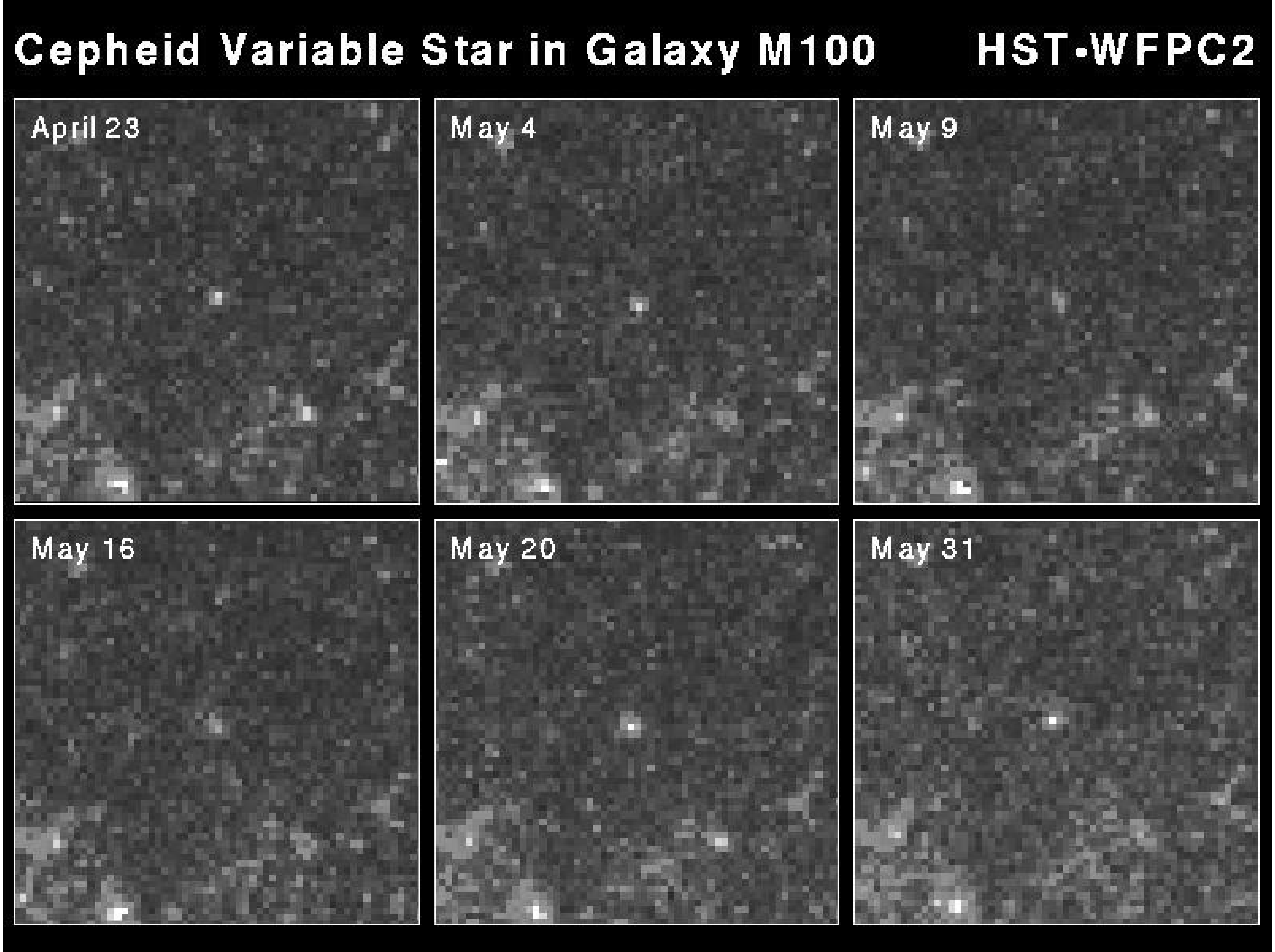
$$\lambda_{observed} = (1 + z)\lambda_{emitted}$$

Observing the Expansion of the Universe

The Light from things which are ***further away*** that we detect ***today*** should have been emitted ***earlier*** in the Universe when the scale factor was ***smaller***.

If the Universe is expanding, redshift should go up with distance, so we need to plot redshift vs. distance. We get the distance from ***knowing*** how bright certain objects are and ***observing*** how bright they appear. We call these “standard candles”.

e.q. Cepheid Variables.


Cepheid Variable Star in Galaxy M100
HST•WFPC2
April 23
May 4
May 9
May 16
May 20
May 31

Observing the Expansion of the Universe

The Light from things which are ***further away*** that we detect ***today*** should have been emitted ***earlier*** in the Universe when the scale factor was ***smaller***.

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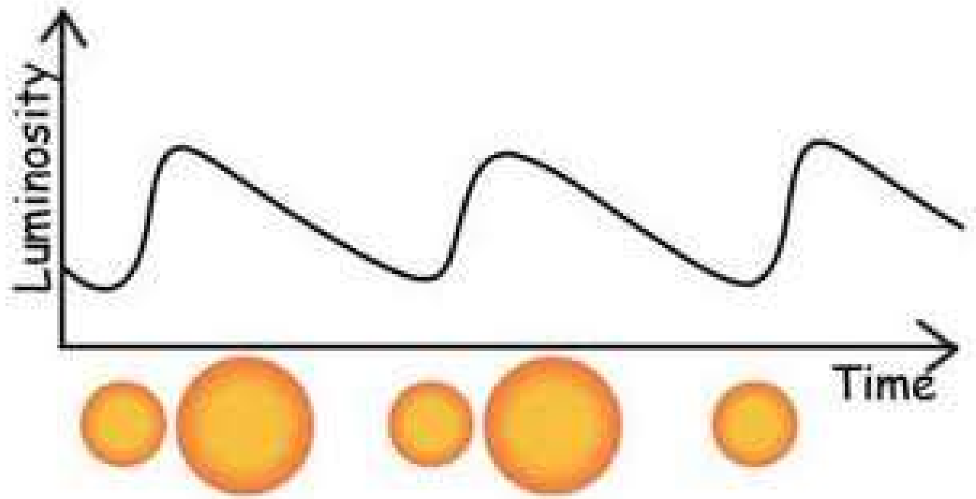
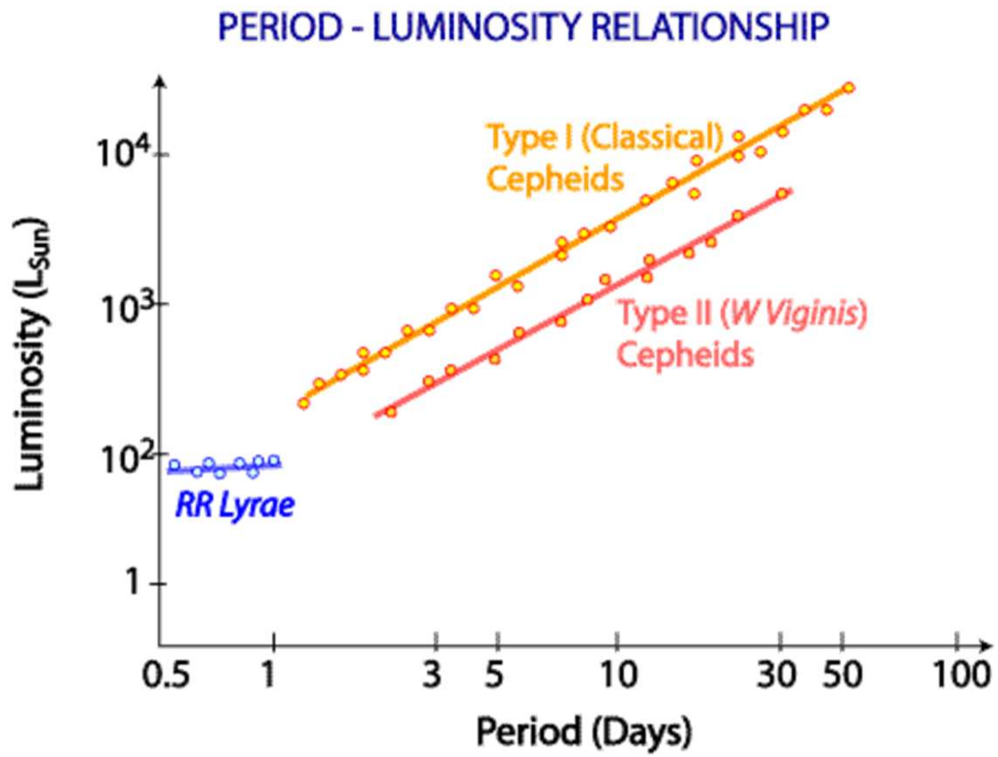
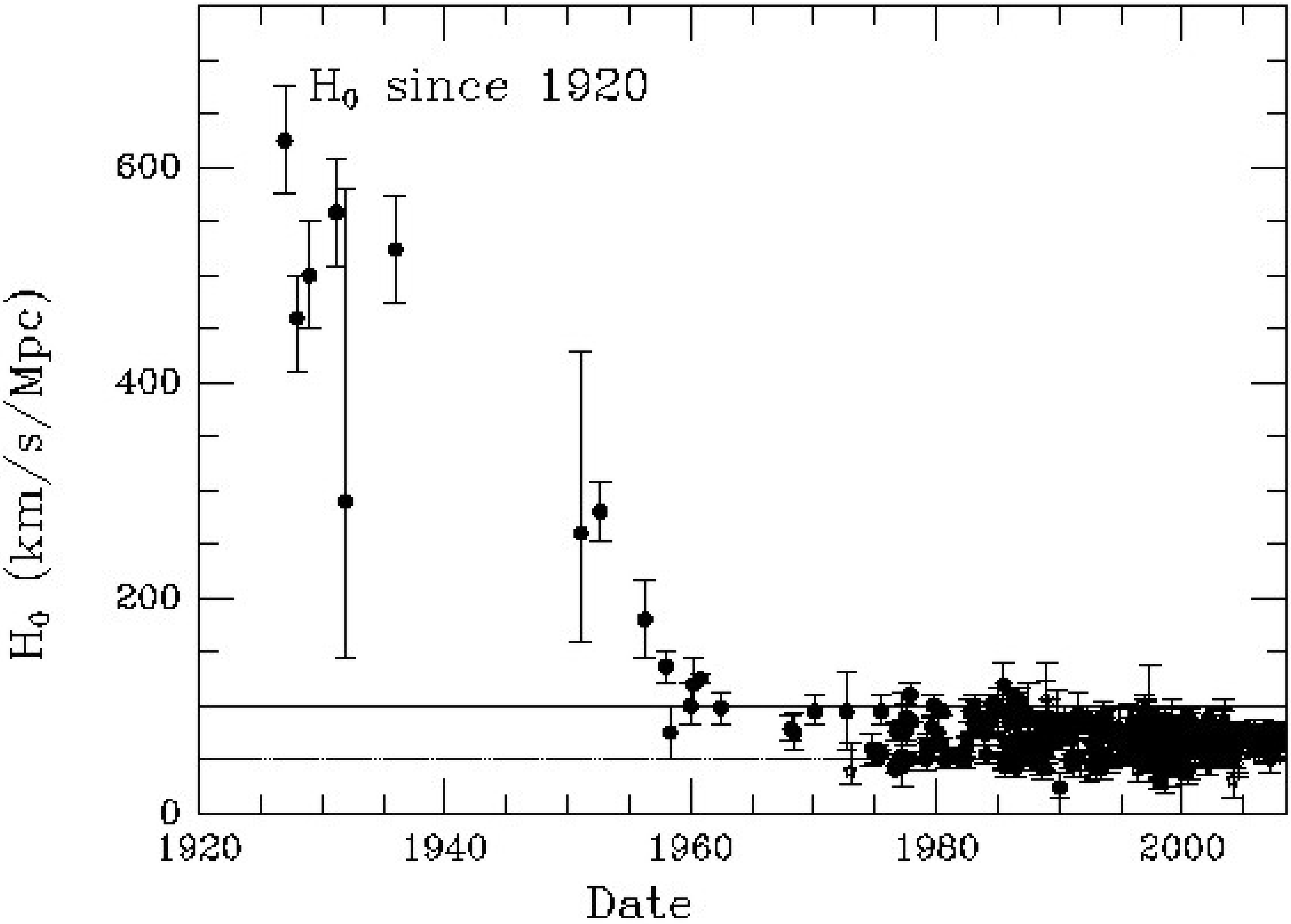
e.q. Cepheid Variables.   Pulsating stars which get bigger and smaller. If they pulsate at a certain rate, we know how bright they are. This is what Hubble used.

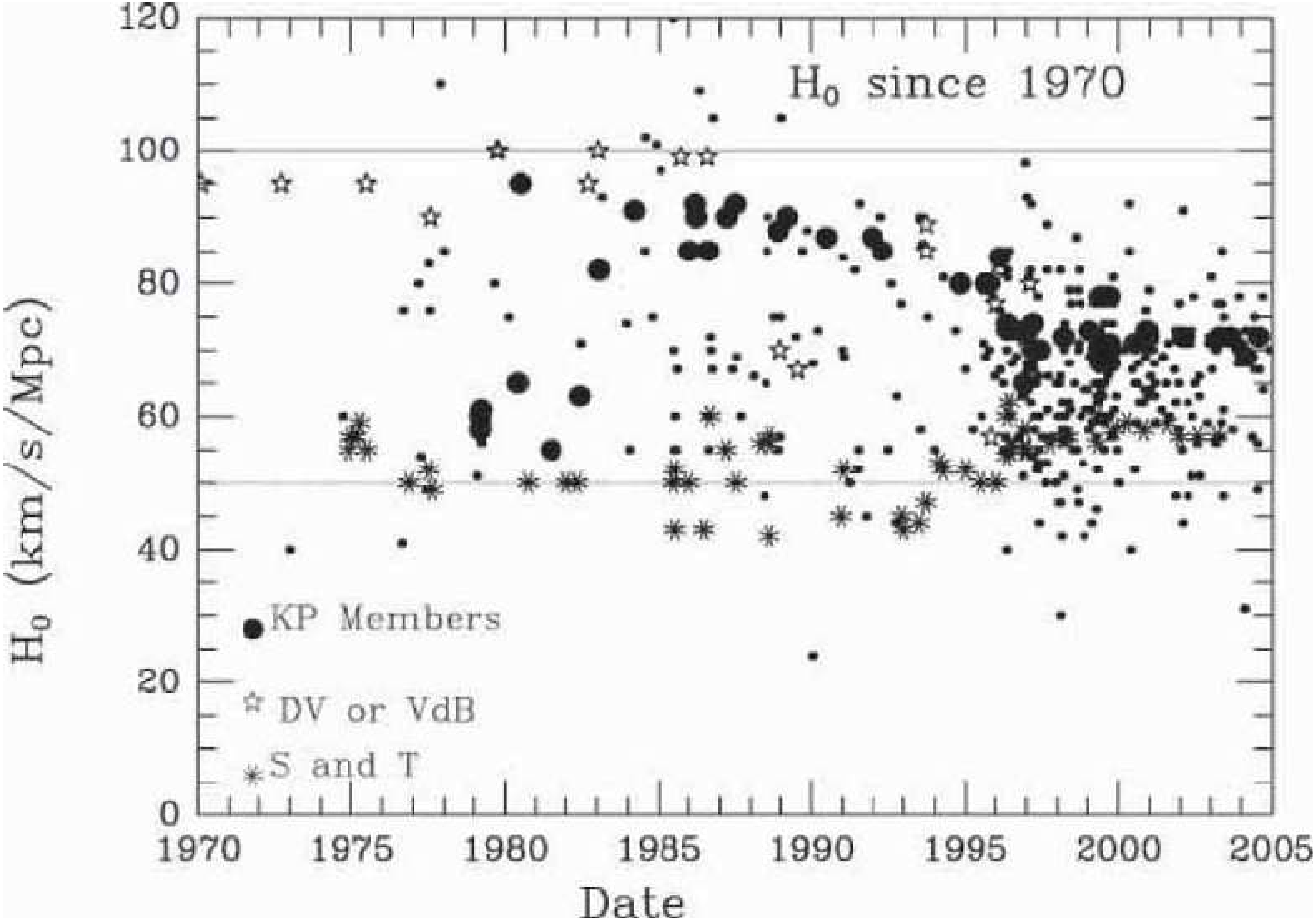


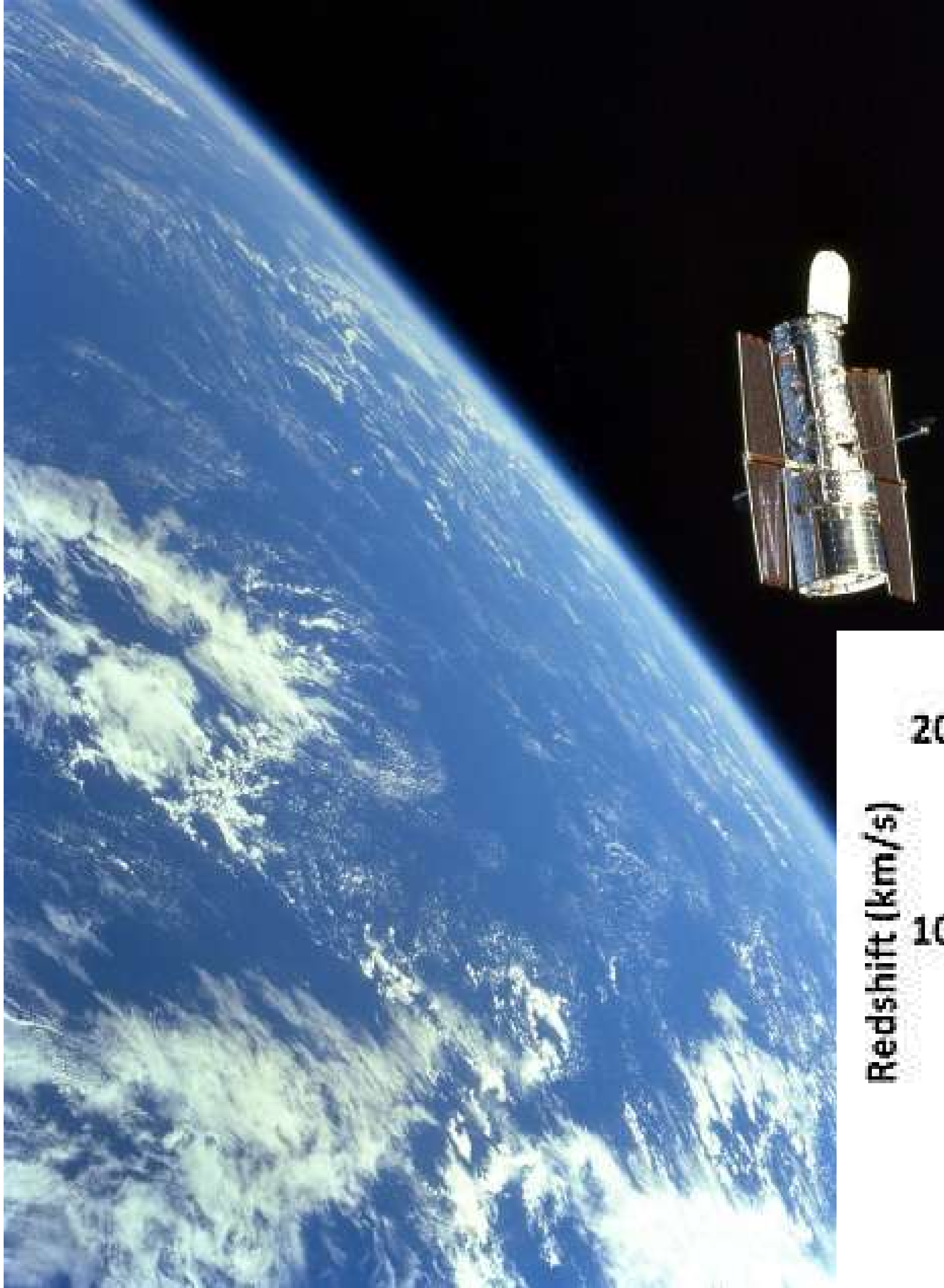




H$_0$ since 1920
H$_0$ (km/s/Mpc)
Date
0
200
400
600
1920
1940
1960
1980
2000

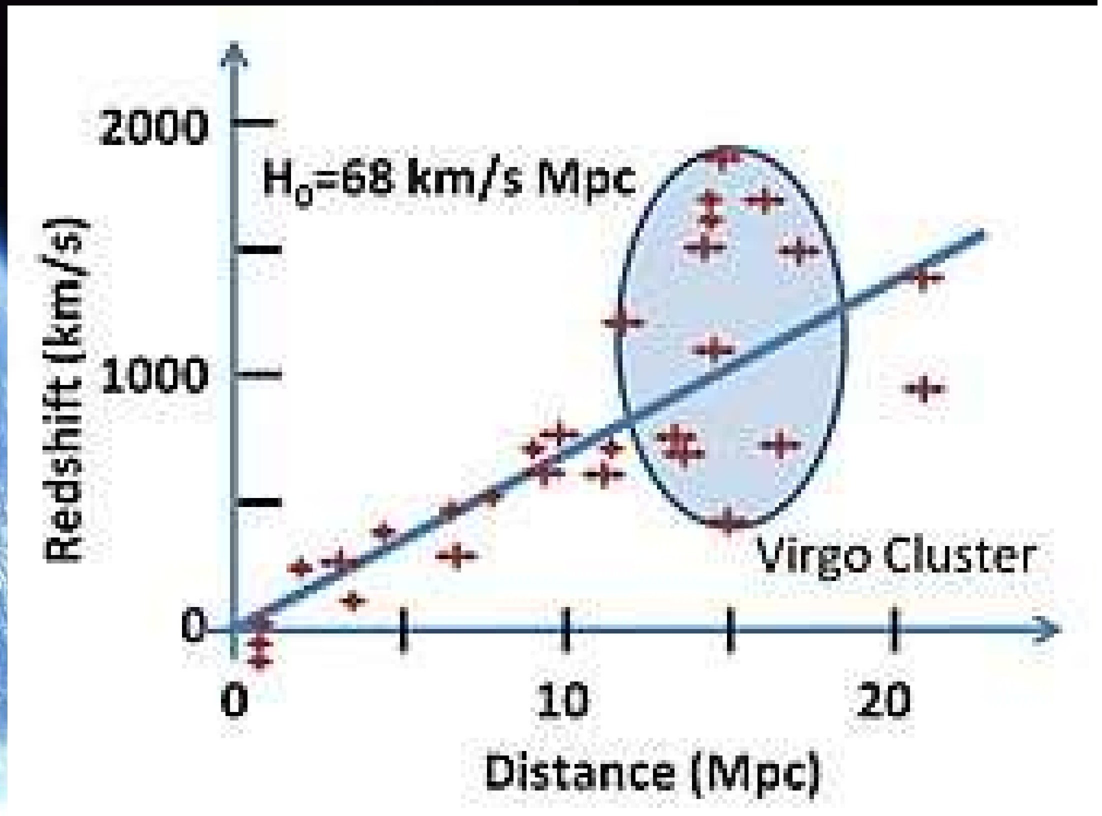
Quite difficult to measure H_0

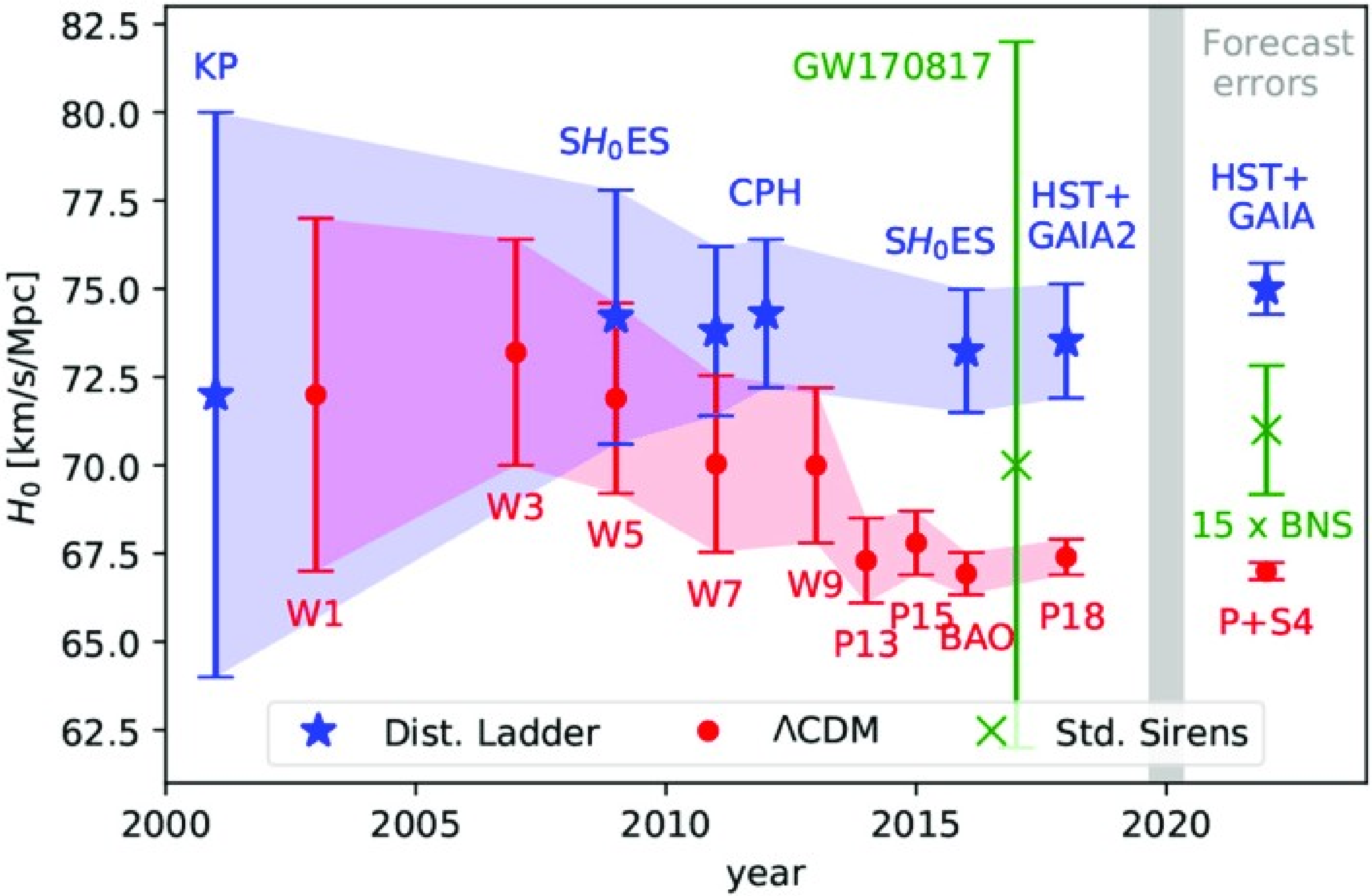




Measurement of H_0 was one of the primary goals of the Hubble Space Telescope

H_0 ~ 70 km / s / Mpc





82.5
80.0
77.5
75.0
72.5
70.0
67.5
65.0
62.5
H_0 [km/s/Mpc]
2000
2005
2010
2015
2020
year
KP
W1
W3
SH0ES
W5
W7
CPH
W9
P13
P15
BAO
SH0ES
GW170817
HST+ GAIA2
P18
Forecast errors
HST+ GAIA
15 x BNS
P+S4
Dist. Ladder
ΛCDM
Std. Sirens

What is Ω ?

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2}$$

If we measure the Hubble rate H, then we know precisely the value of ρ that we would need in order for k to be equal to zero. We call this the critical density ρ_{crit}

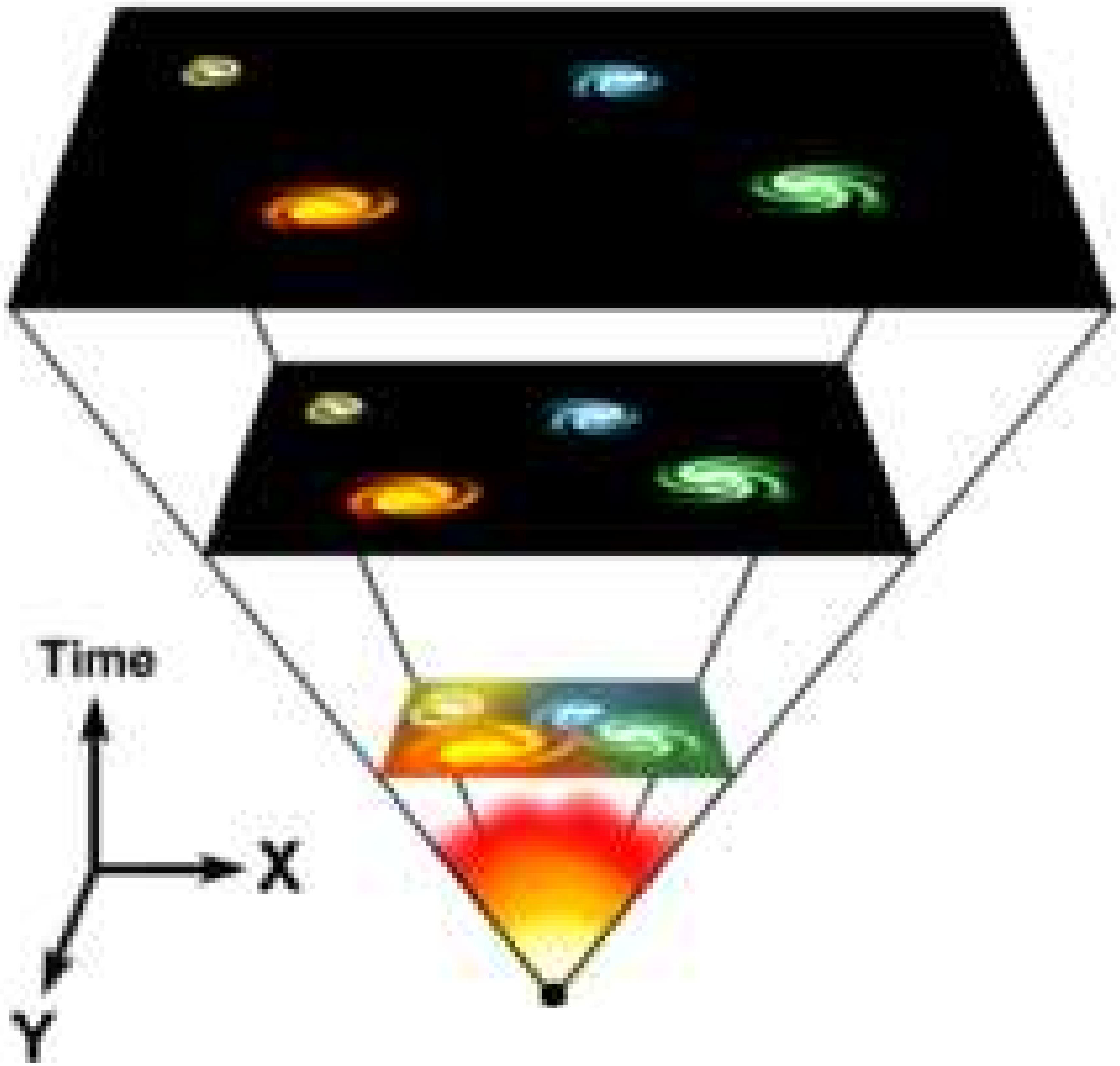
$$\rho_{crit} = \frac{3H_0^2}{8\pi G}$$

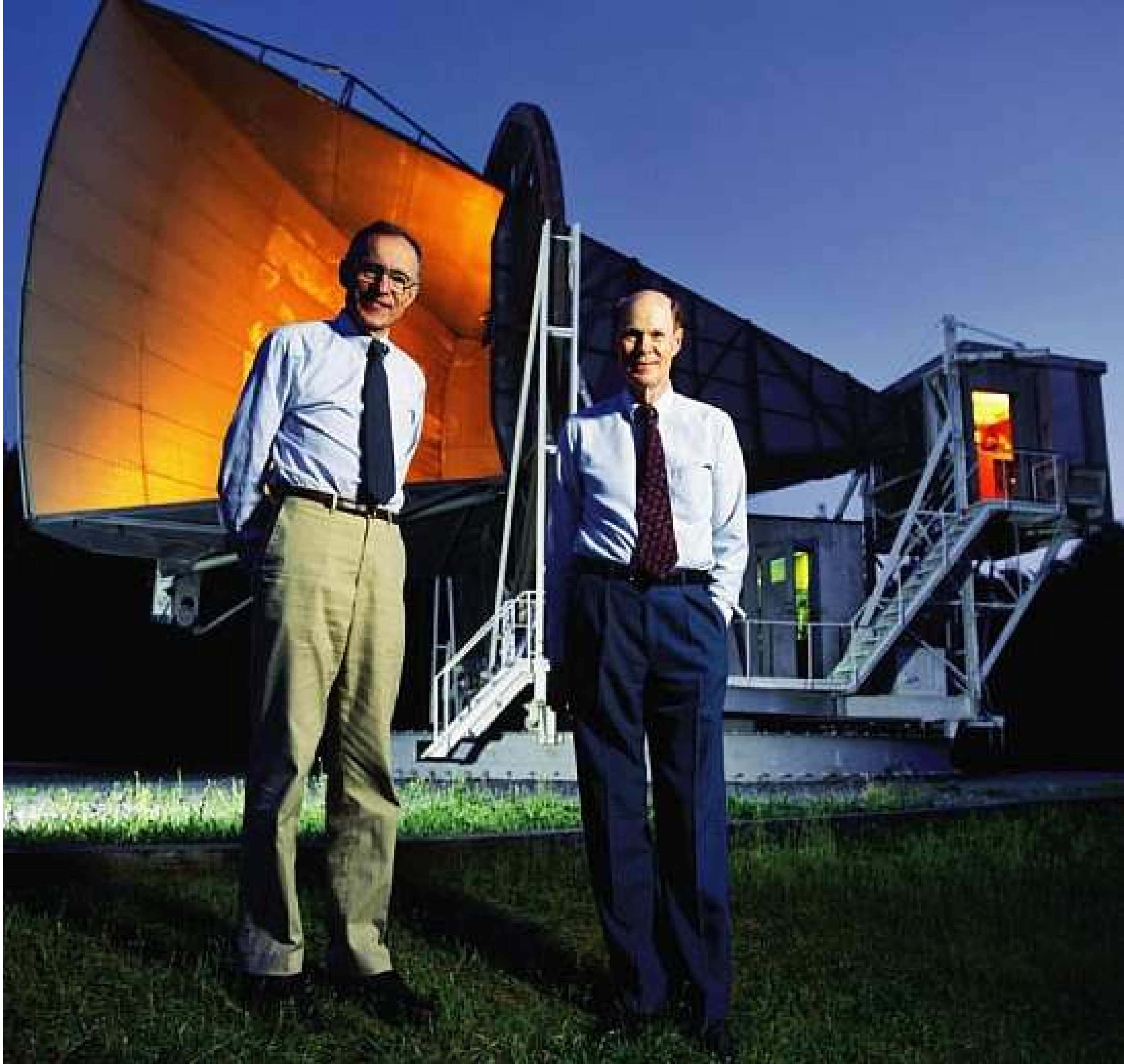
and

$$\Omega = \frac{\rho}{\rho_{crit}}$$

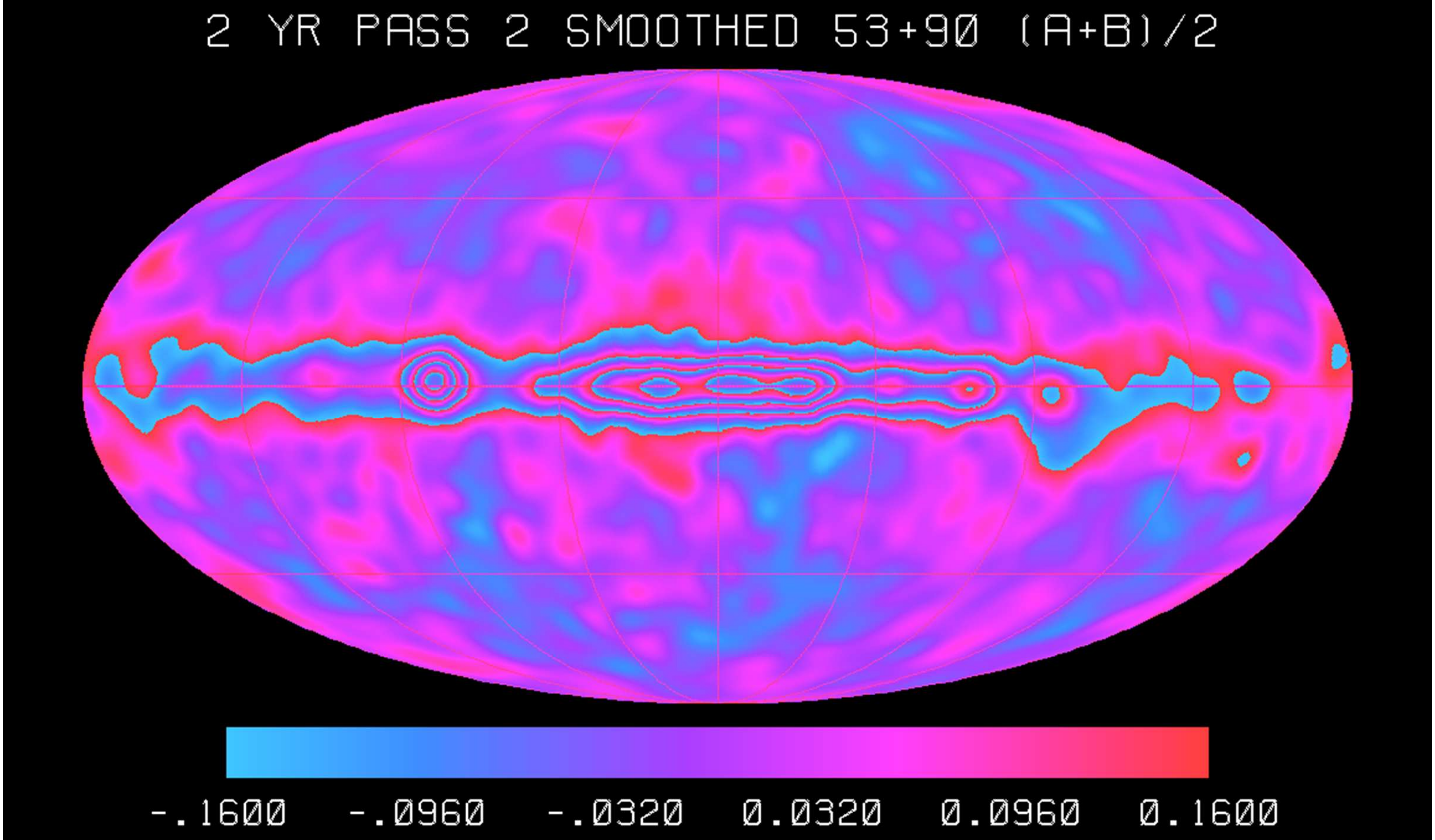
So that if $\rho<\rho_{crit}$ then $\Omega<1$ and the Universe is negatively curved (spatial curvature).

Alternatively if $\rho>\rho_{crit}$ then $\Omega>1$ and the Universe is positively curved.


Time
X
Y



Penzias
and
Wilson


2 YR PASS 2 SMOOTHED 53+90 (A+B)/2
-.1600
-.0960
-.0320
0.0320
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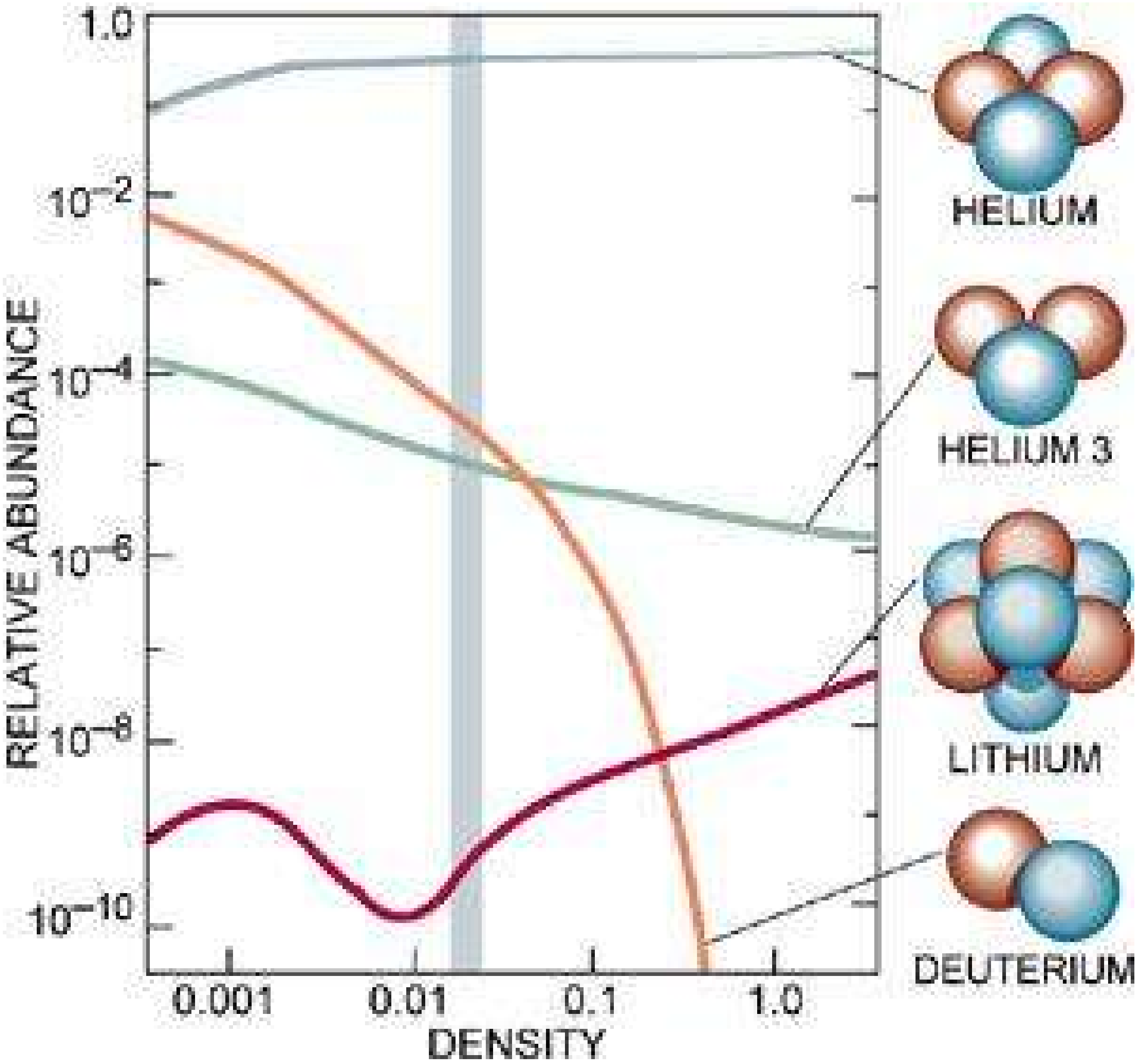
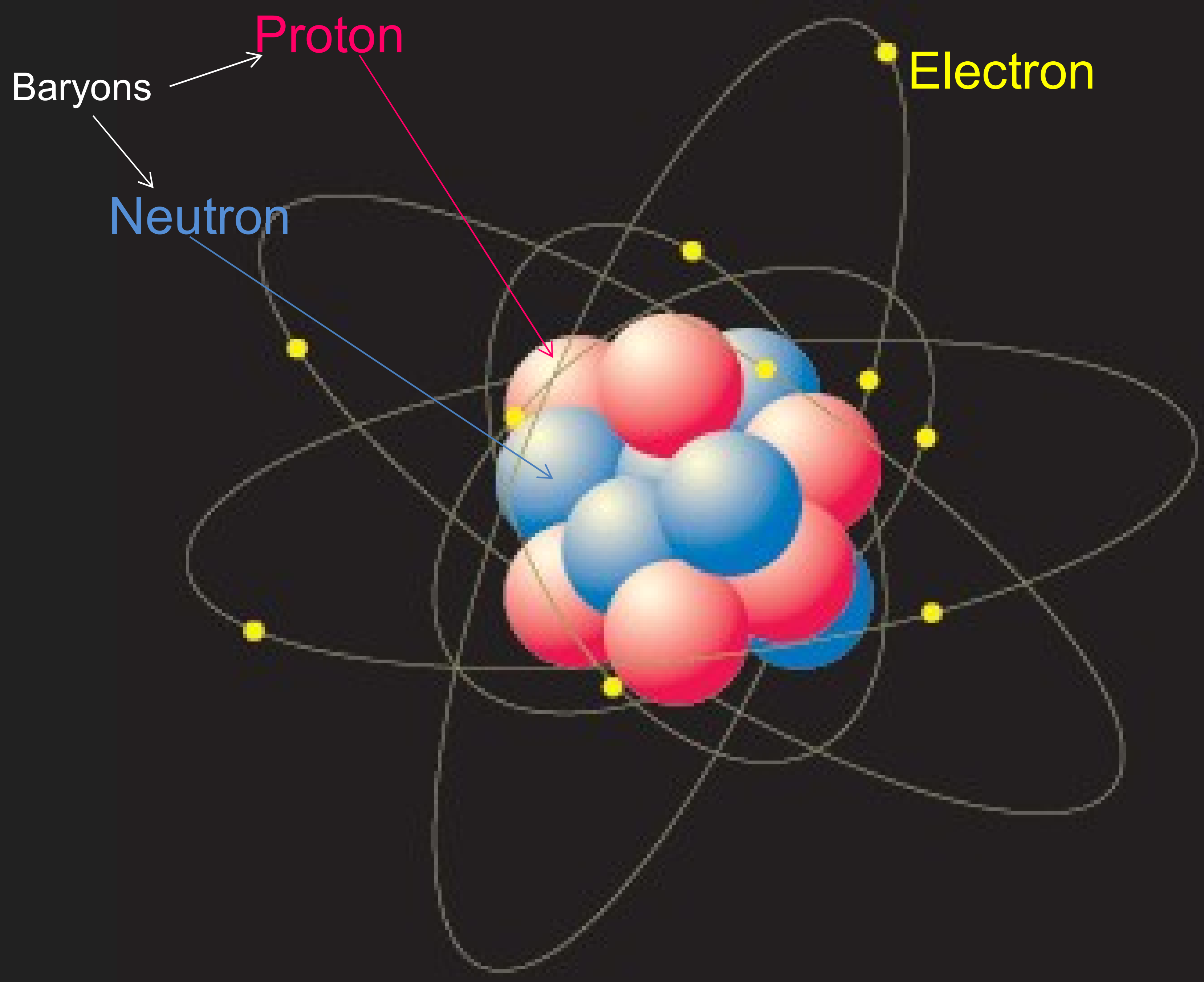
0.1600



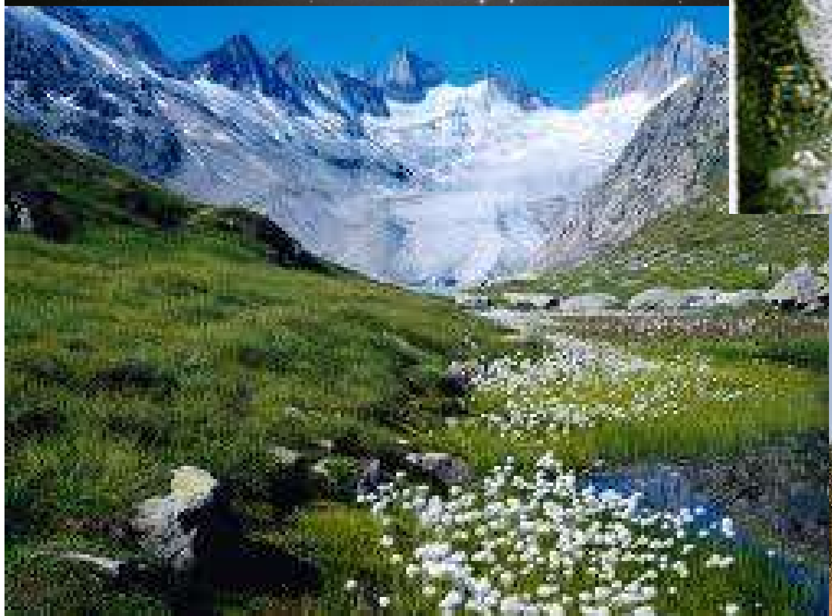
Baryon to photon ratio

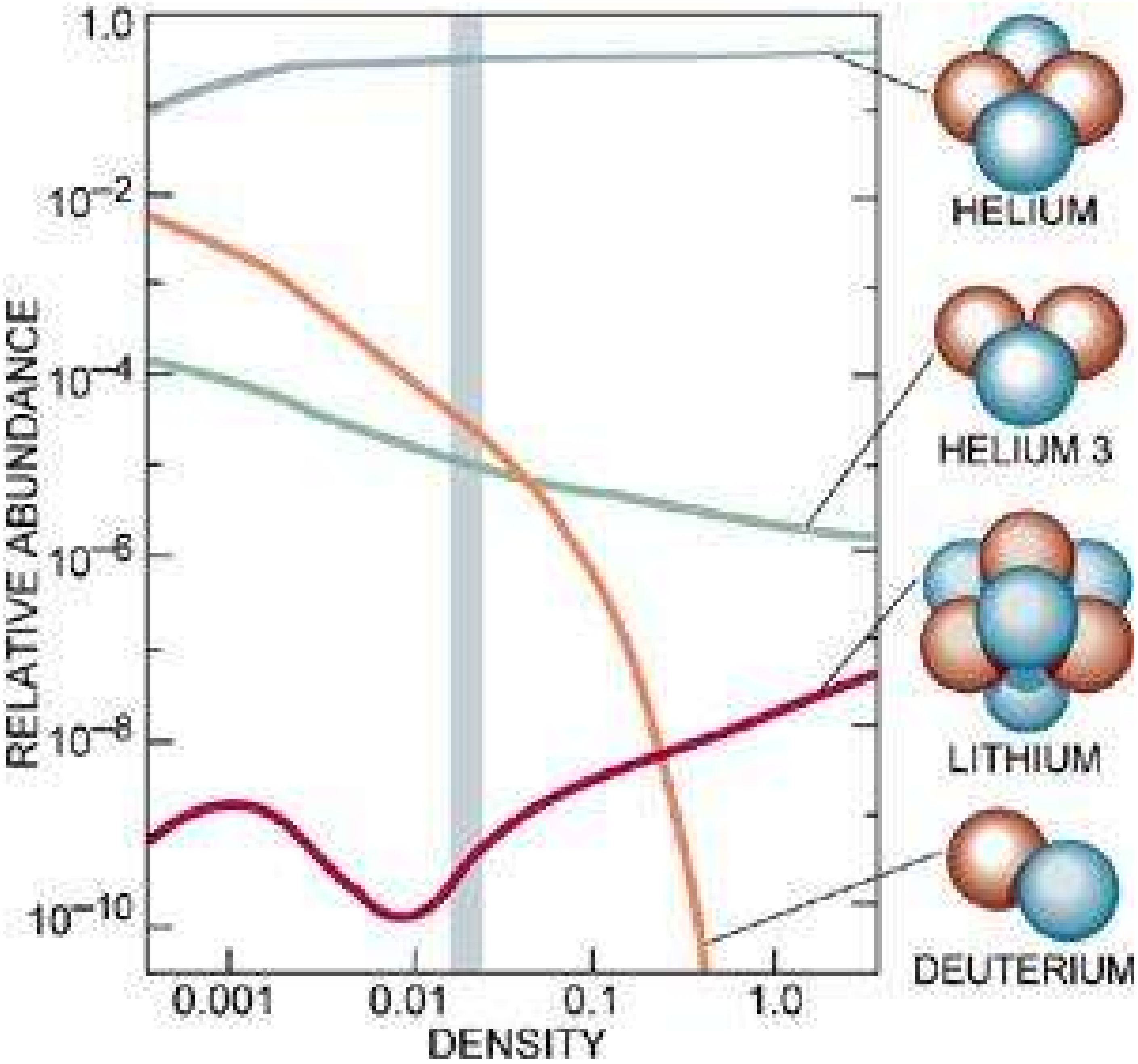
Ratio of light elements gives baryon to photon ratio



Mass of baryons much bigger than mass of protons

So when cosmologists refer to “Baryons” they mean “everything”





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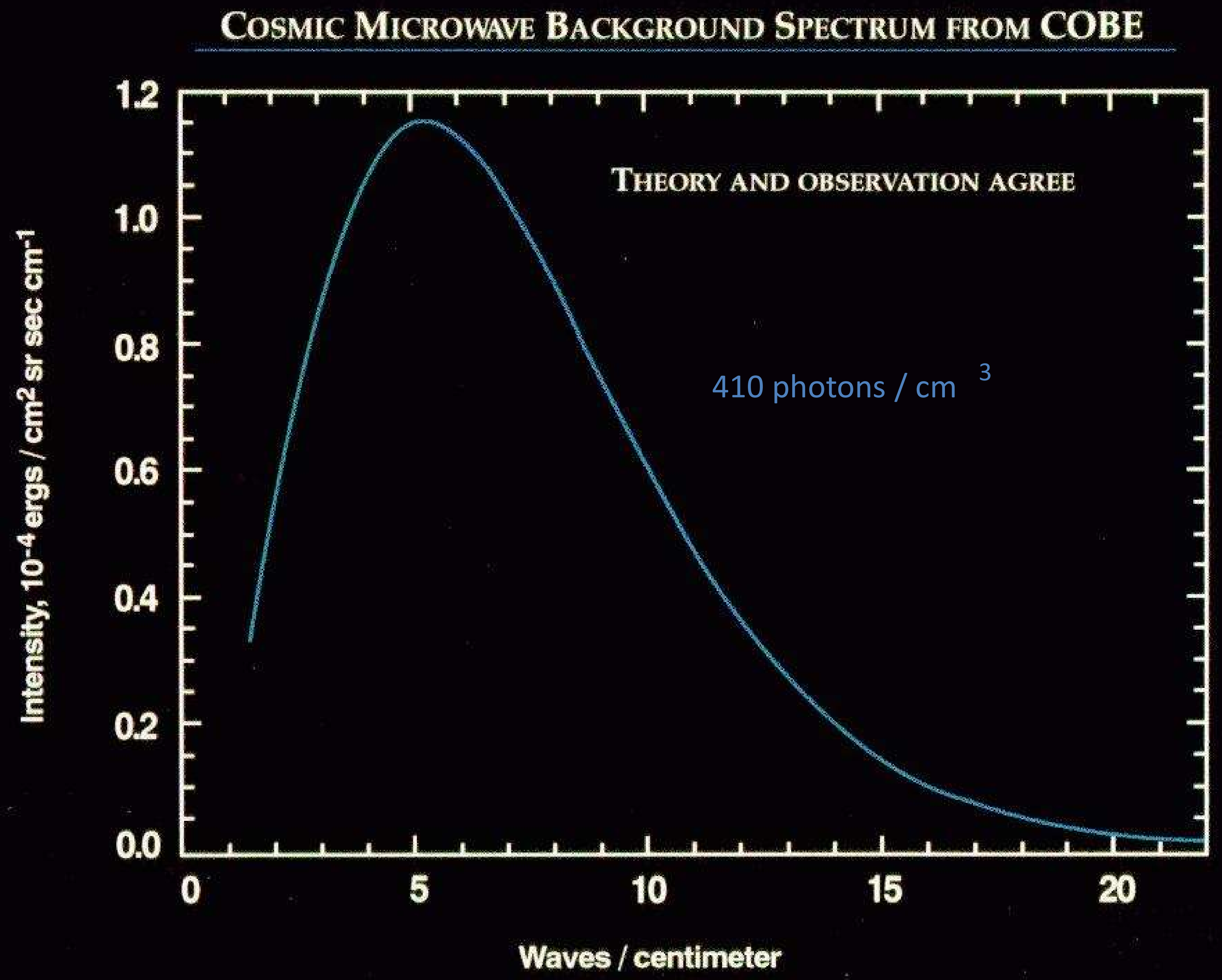
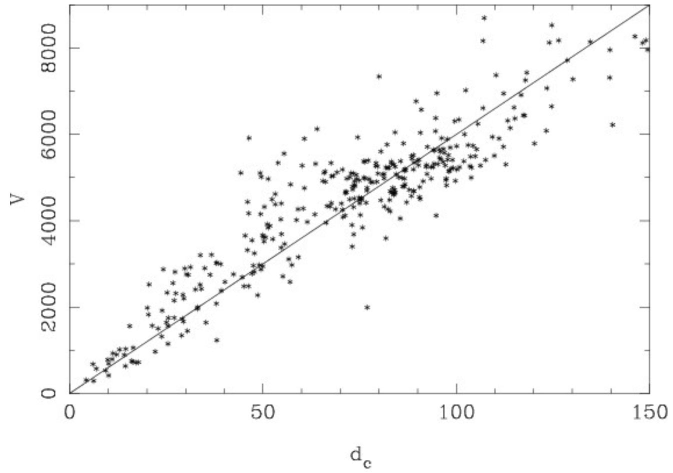
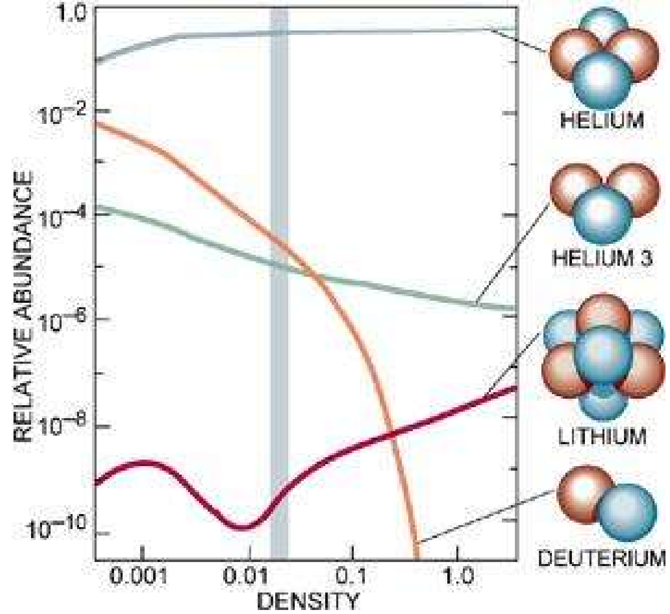

COSMIC MICROWAVE BACKGROUND SPECTRUM FROM COBE
THEORY AND OBSERVATION AGREE
410 photons / cm 3
Intensity, 10-4 ergs / cm2 sr sec cm-1
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
Waves / centimeter





$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3}$$

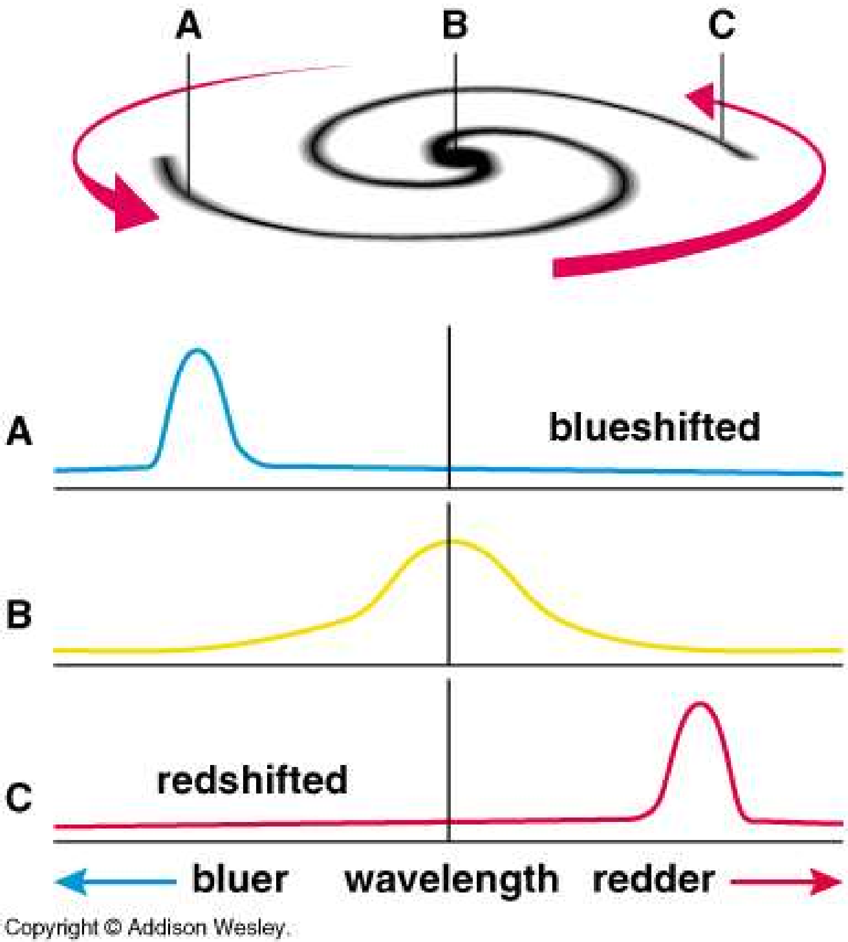
$10^{-29}\ \mathrm{g\ cm^{-3}}$

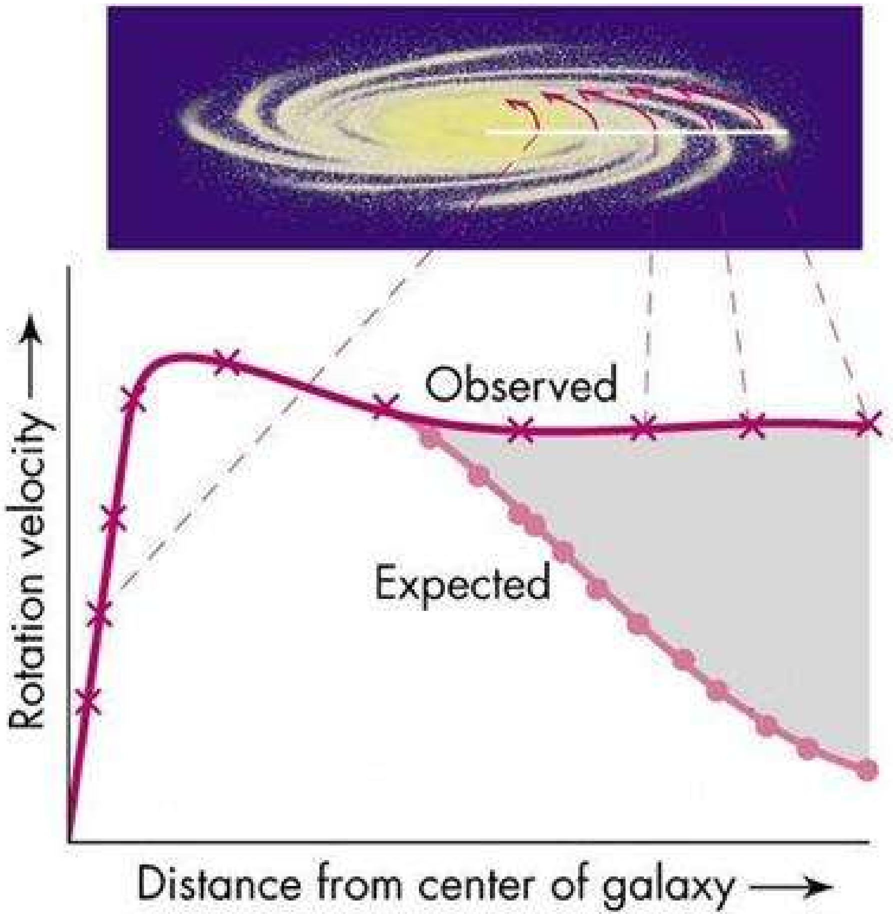
$4 \times 10^{-31}\mathrm{g\ cm^{-3}}$

What's all the rest???

"When I was in school, I was continually told to go find something else to study, or told I wouldn't get a job as an astronomer. But I just didn't listen. If it's something you really want to do, you just have to do it—and maybe have the courage to do it a little differently."

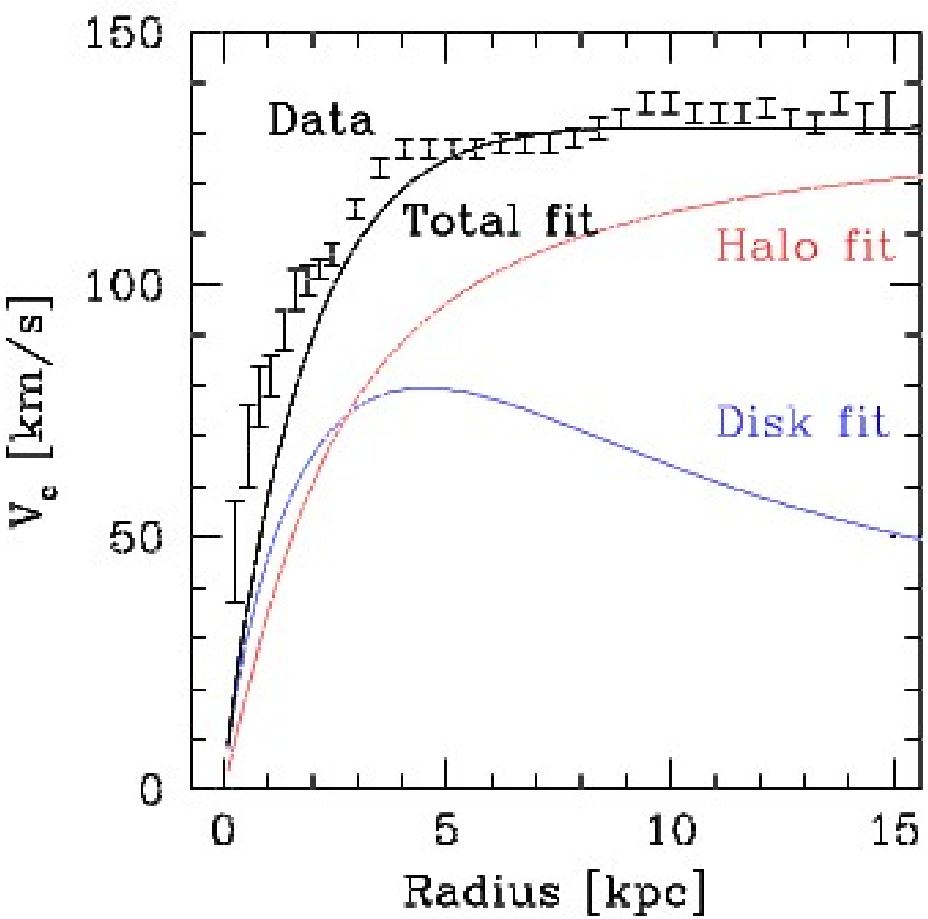
Flat rotation curves – Vera Rubin 1970

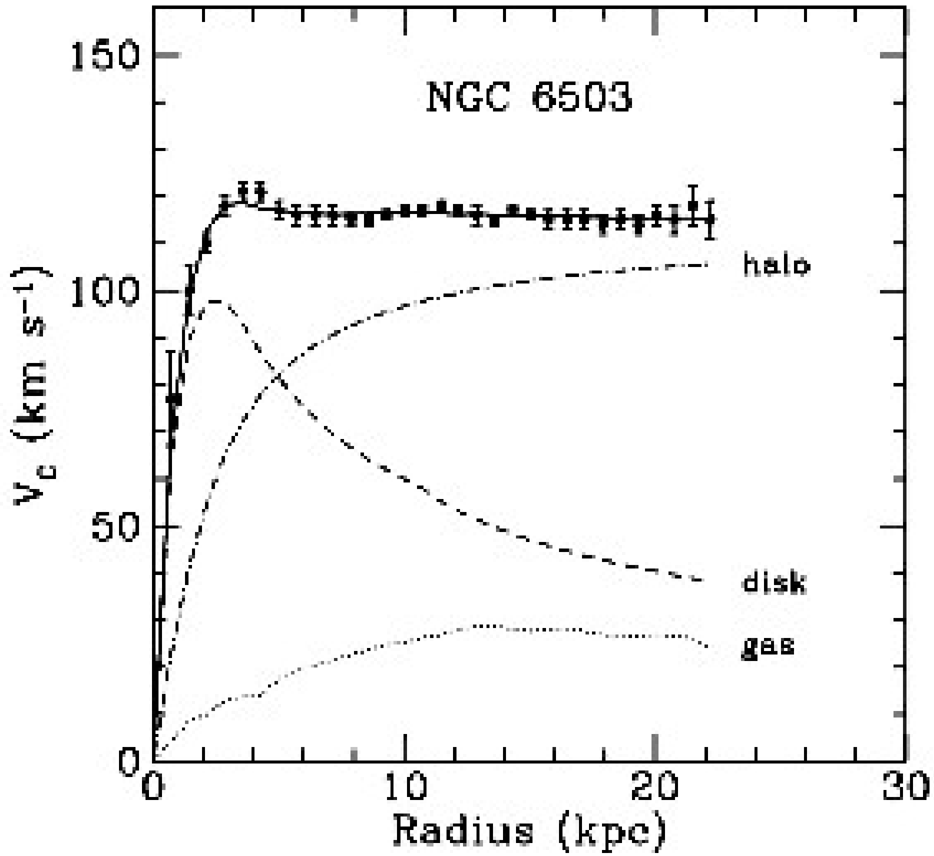




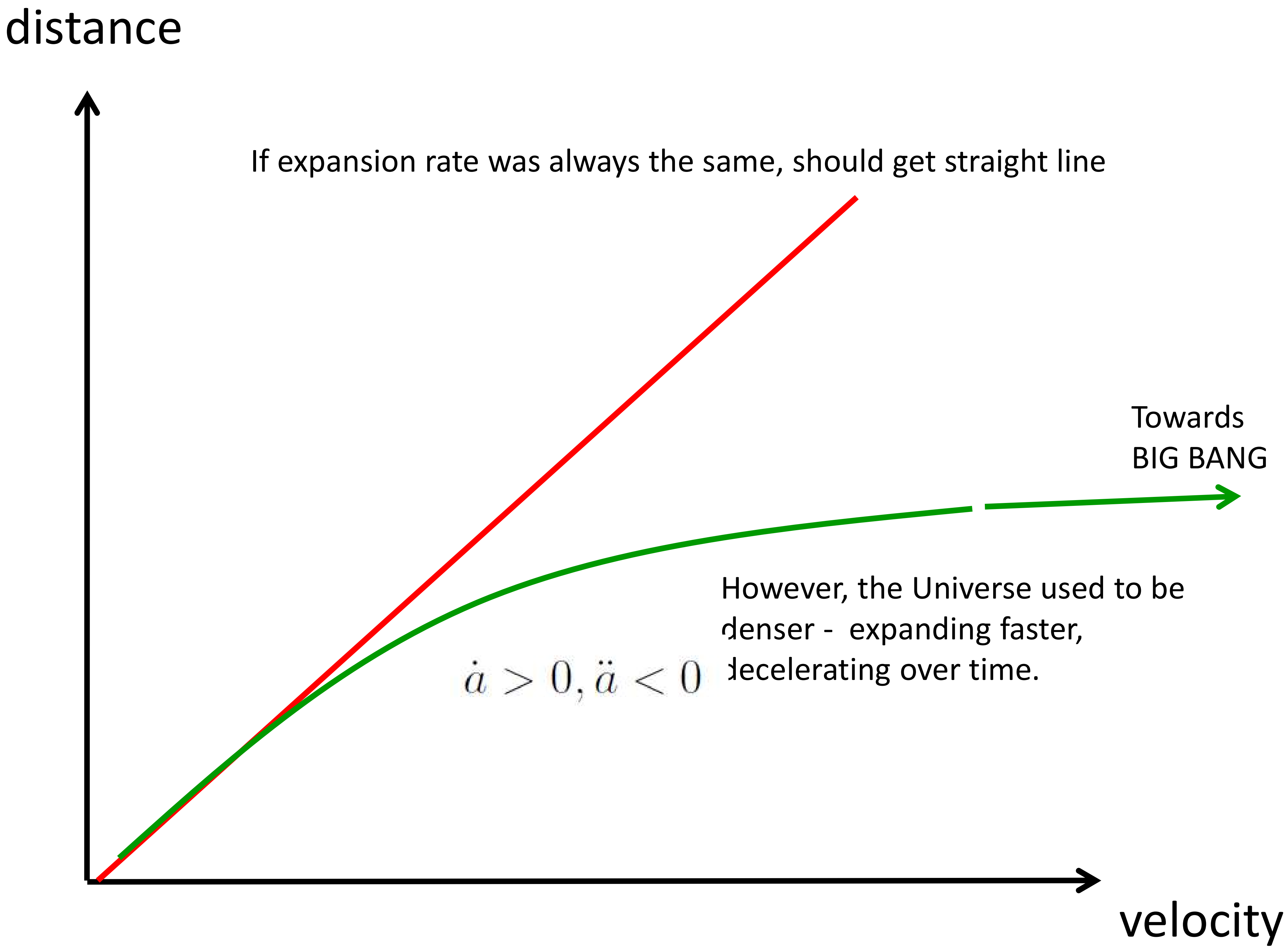
HI velocity field of NGC 5055

More Examples of flat rotation curves

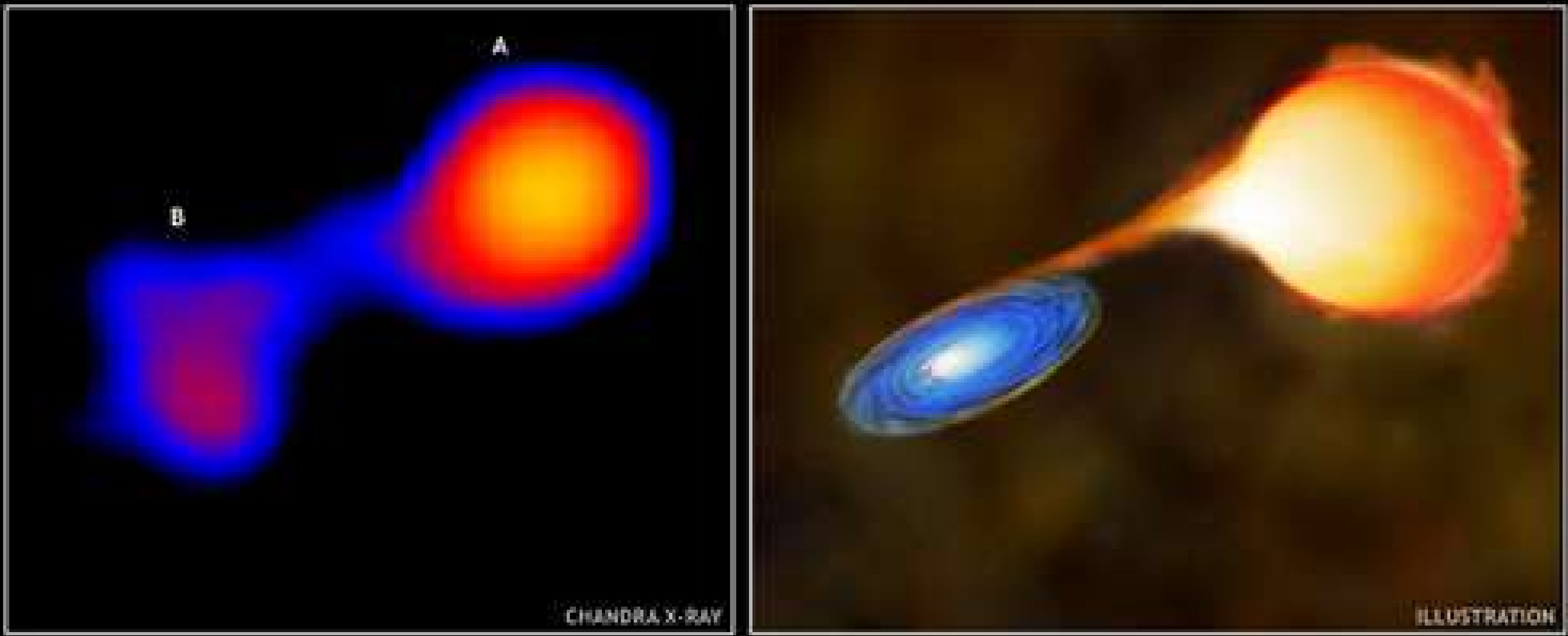




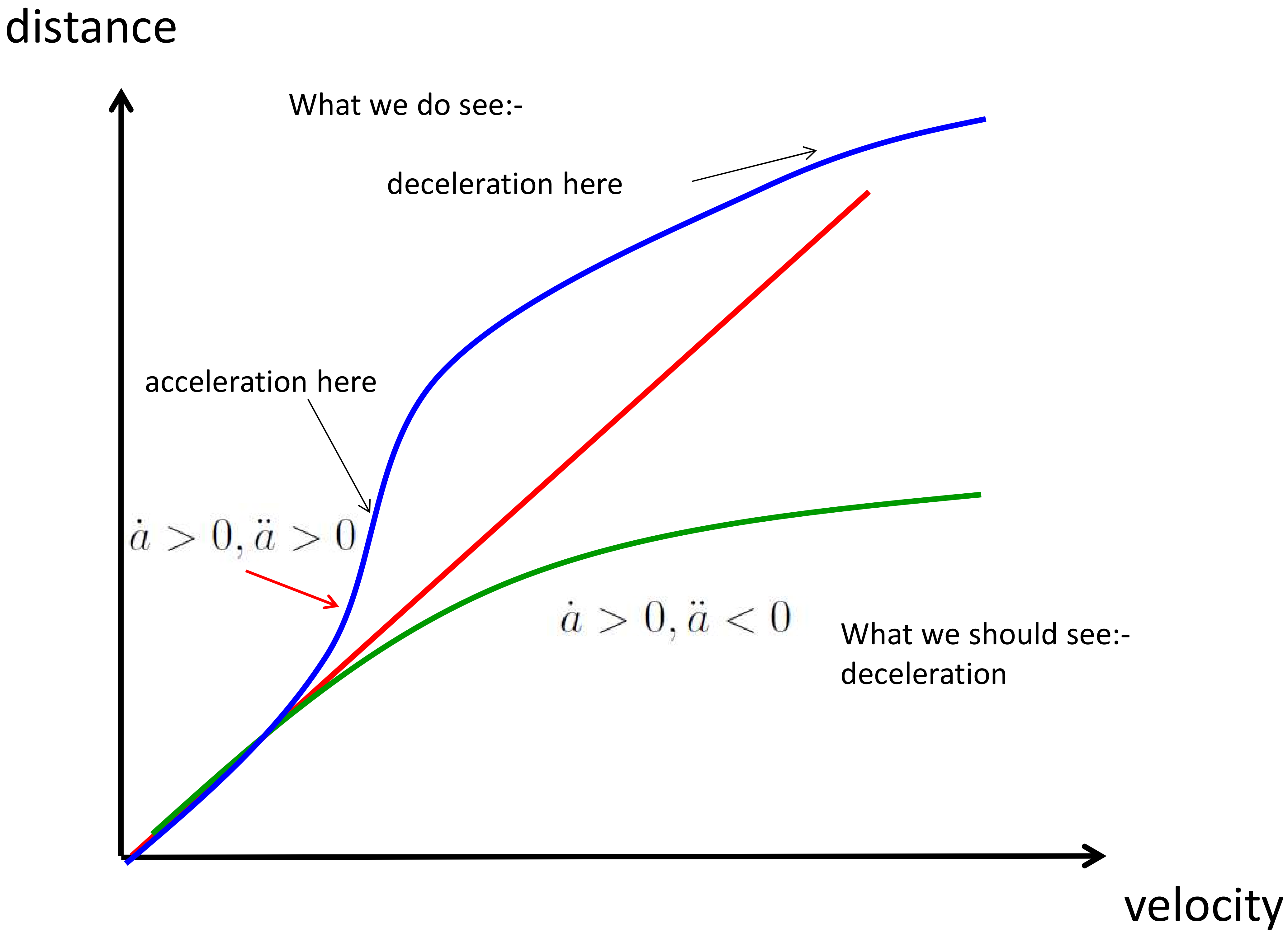
$$v^2 = \frac{Gm_1}{r}$$


distance
If expansion rate was always the same, should get straight line
Towards
BIG BANG
However, the Universe used to be
denser - expanding faster,
decelerating over time.
$\dot{a} > 0, \ddot{a} < 0$
velocity

Type 1a supernovae as Standard candles



Brighter, type 1a are always roughly the same brightness.
Allow us to see much further back than cepheid variables.



distance
What we do see:-
deceleration here
acceleration here
$\dot{a} > 0, \ddot{a} > 0$
$\dot{a} > 0, \ddot{a} < 0$
What we should see:-
deceleration
velocity

Acceleration implies negative pressure

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

To get positive acceleration we need $P < -\rho/3$

In cosmology, pressure tells you how fast the density of something decreases as the Universe expands

$$\dot{\rho} = -3H(\rho + P)$$

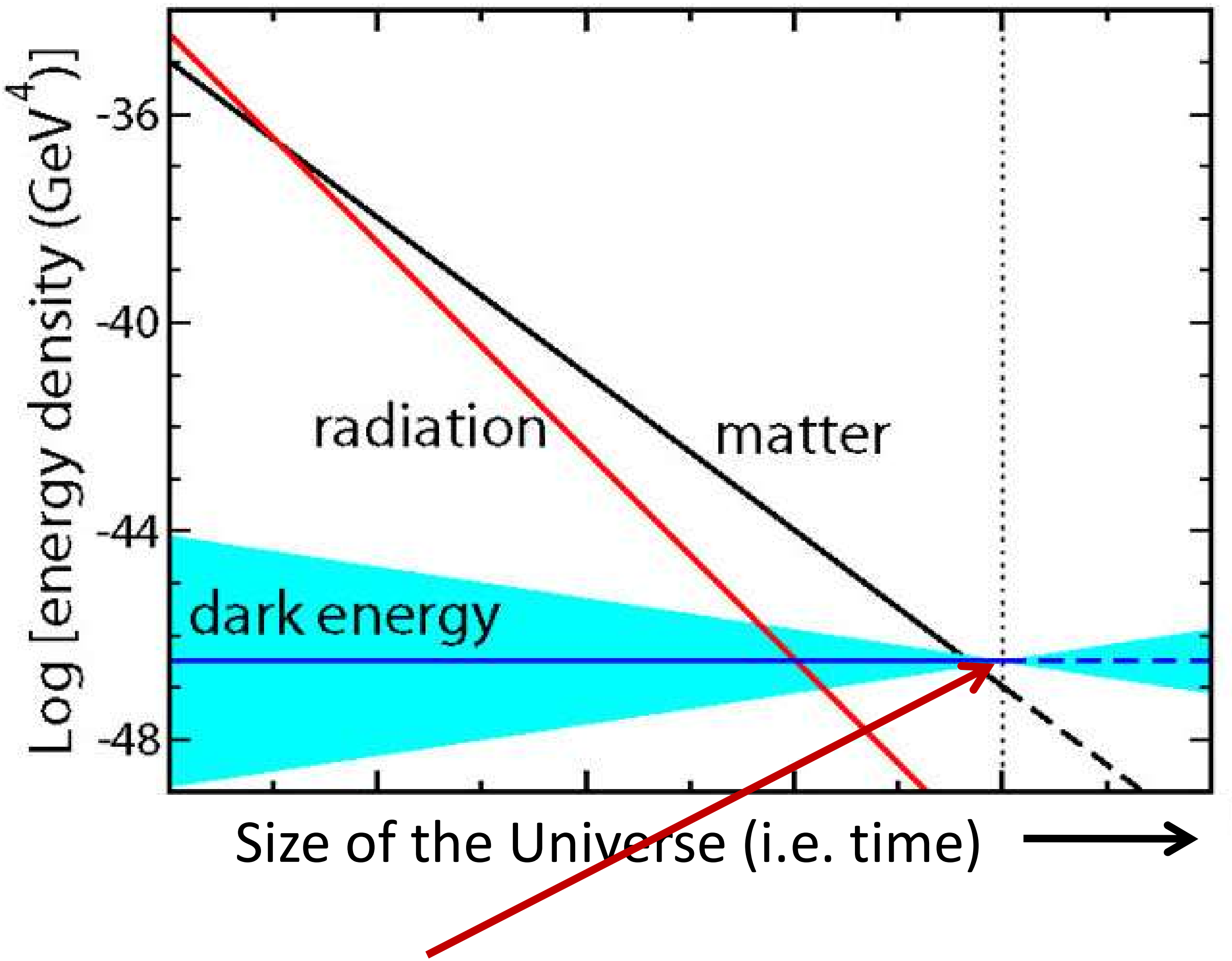
Friedman Equation with Cosmological Constant

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3}$$

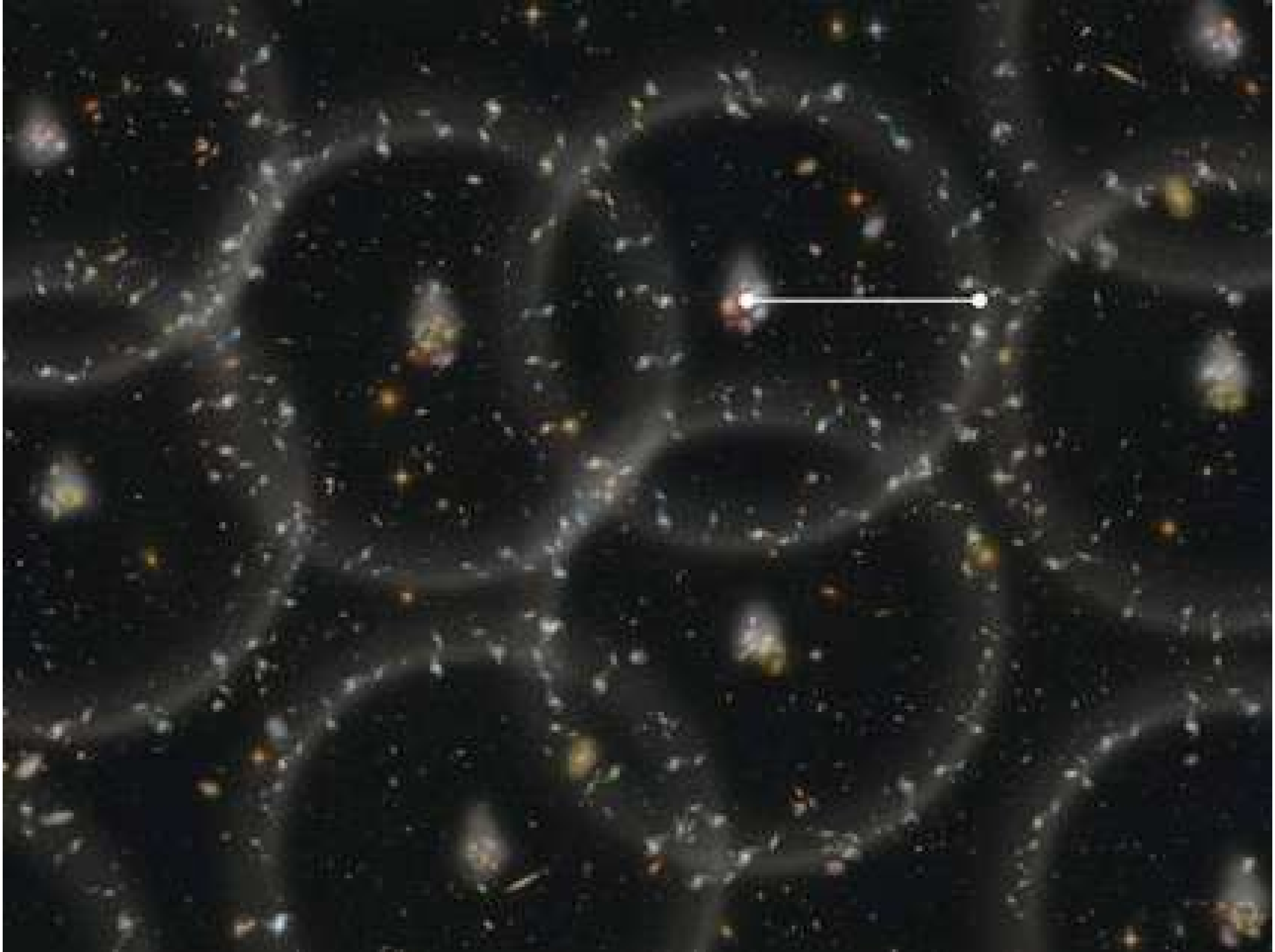
$$= \frac{8\pi G(\rho + \rho_\Lambda)}{3} - \frac{k}{a^2}$$

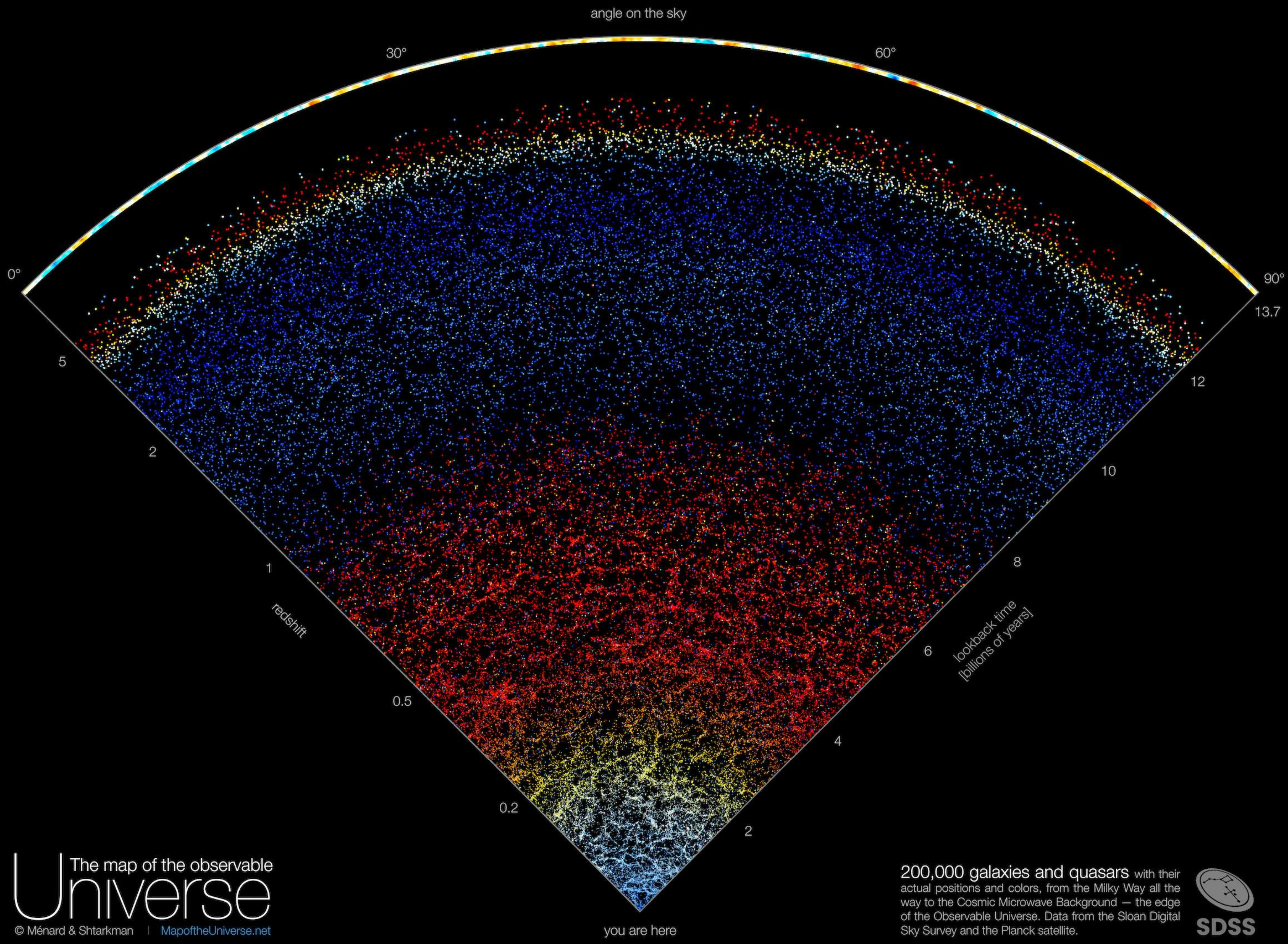
We can think of cosmological constant as a constant energy density

Different energies and how they dilute



Why are we here? (cosmic coincidence problem)

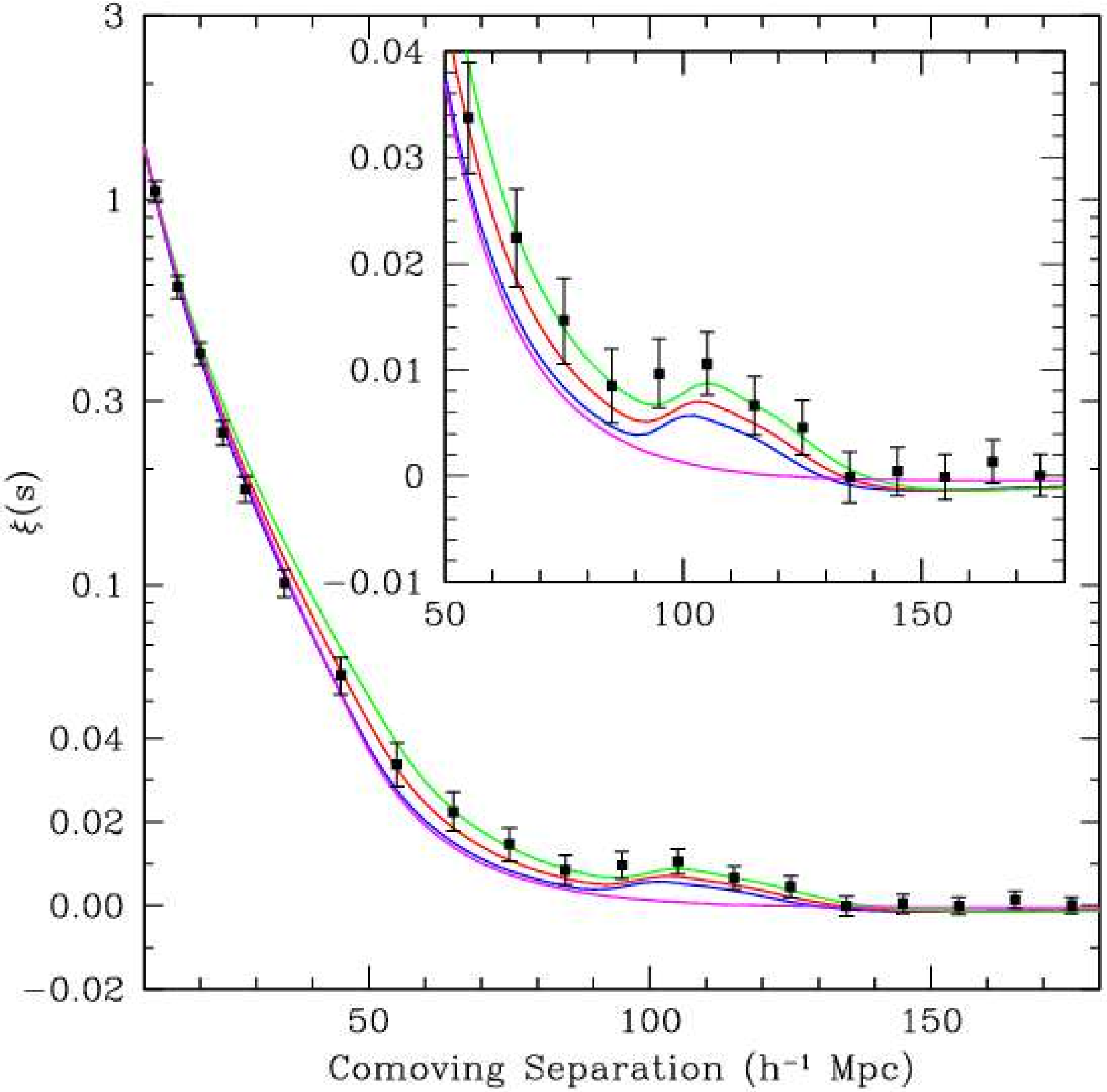




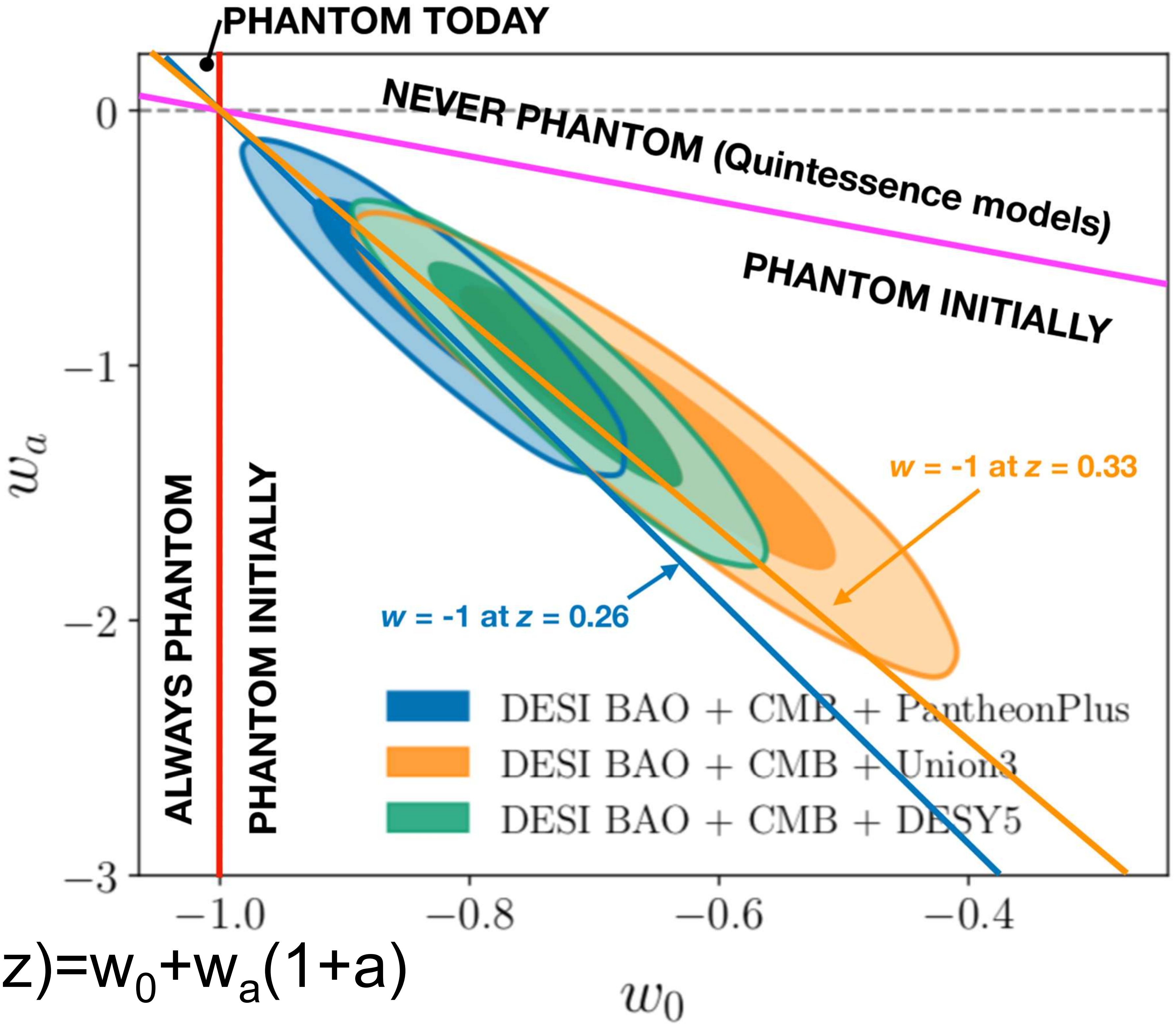
The map of the observable Universe

200,000 galaxies and quasars with their actual positions and colors, from the Milky Way all the way to the Cosmic Microwave Background — the edge of the Observable Universe. Data from the Sloan Digital Sky Survey and the Planck satellite.

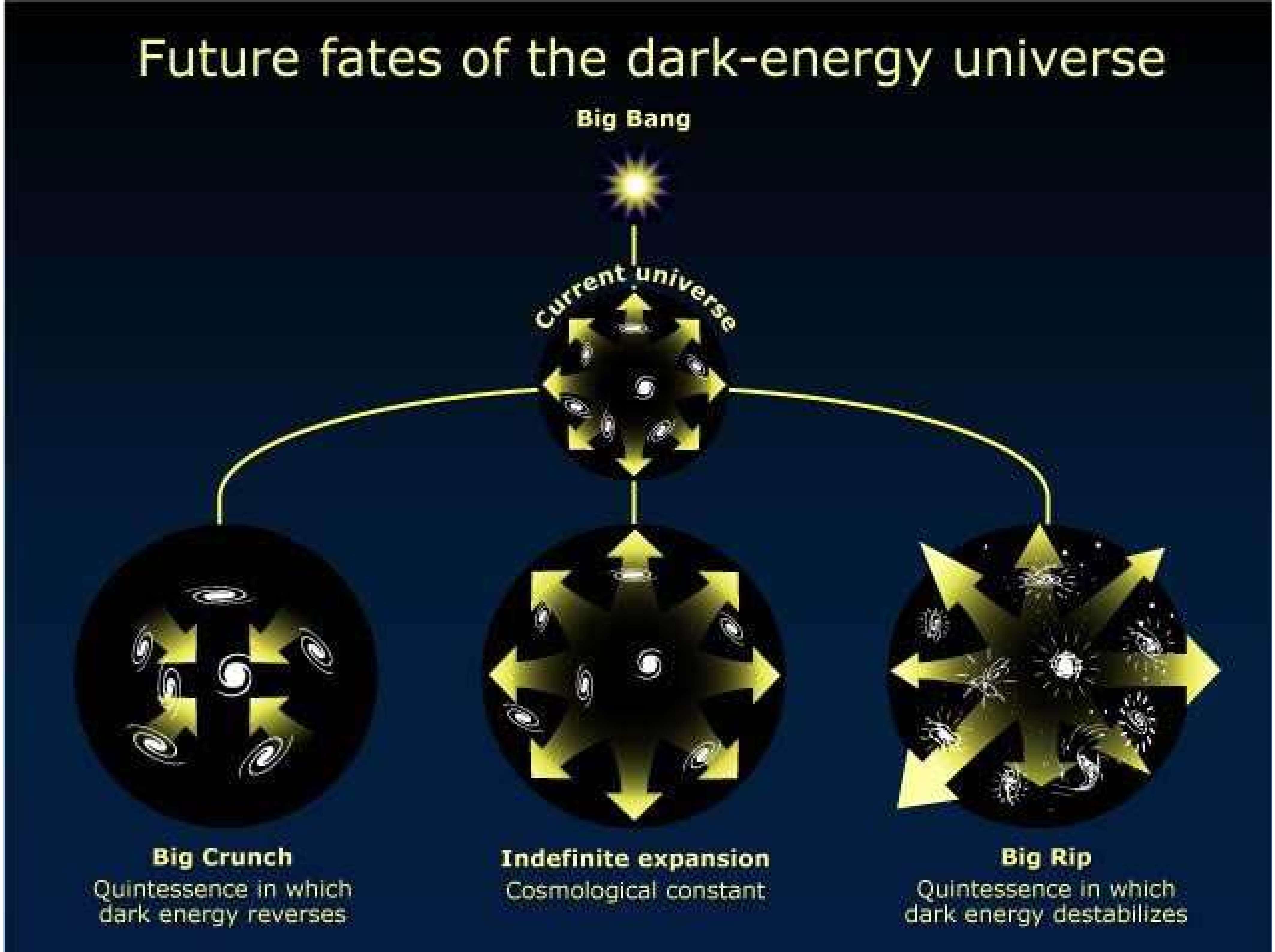
SDSS



3
1
0.3
0.1
0.04
0.02
0.00
−0.02
50
100
150
ξ(s)
Comoving Separation (h⁻¹ Mpc)
0.04
0.03
0.02
0.01
0
−0.01
50
100
150


PHANTOM TODAY
NEVER PHANTOM (Quintessence models)
PHANTOM INITIALLY
ALWAYS PHANTOM
PHANTOM INITIALLY
w = -1 at z = 0.33
w = -1 at z = 0.26
DESI BAO + CMB + PantheonPlus
DESI BAO + CMB + Union3
DESI BAO + CMB + DESY5
0
−1
−2
−3
−1.0
−0.8
−0.6
−0.4
w_a
w_0

$w(z)=w_0+w_a(1+a)$


Future fates of the dark-energy universe
Big Bang
Current universe
Big Crunch
Quintessence in which dark energy reverses
Indefinite expansion
Cosmological constant
Big Rip
Quintessence in which dark energy destabilizes

1. Fit a straight line to data with errors.

2. Take the Pantheon Supernova Type 1a catalogue and work out how big Ω_Λ is (assume a flat Universe).

3. Work out actual redshifts of galaxies from five photometric observations.

1. Fit a straight line to data with errors.

$$\chi^2 = \sum_i \frac{\left(y_{theory,i} - y_{data,i}\right)^2}{\sigma_i^2}$$

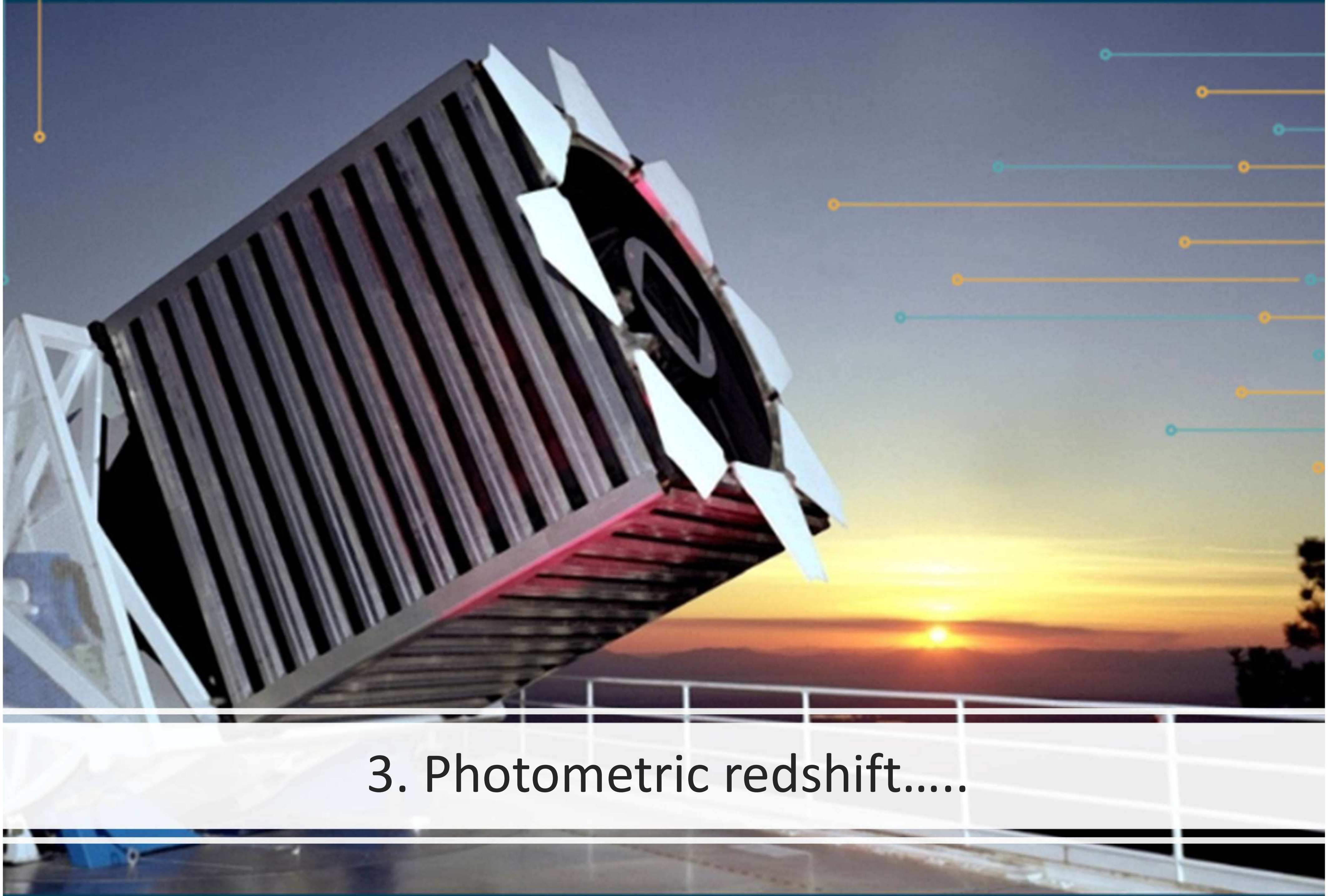
$$y = mx + c$$

I’ve given you x and y data and errors on the y values, so minimise the χ^2 to get the best fit.

2. Take the Pantheon Supernova Type 1a catalogue and work out how big Ω_Λ is (assume a flat Universe).

Read carefully section 7 of the notes provided. There is more detail in section 6 about luminosity distance if you want them.

You don’t know H0 and you don’t know the absolute magnitudes of the supernovae, so you have to treat them as a nuisance parameter that you minimise over.



3. Photometric redshift…..

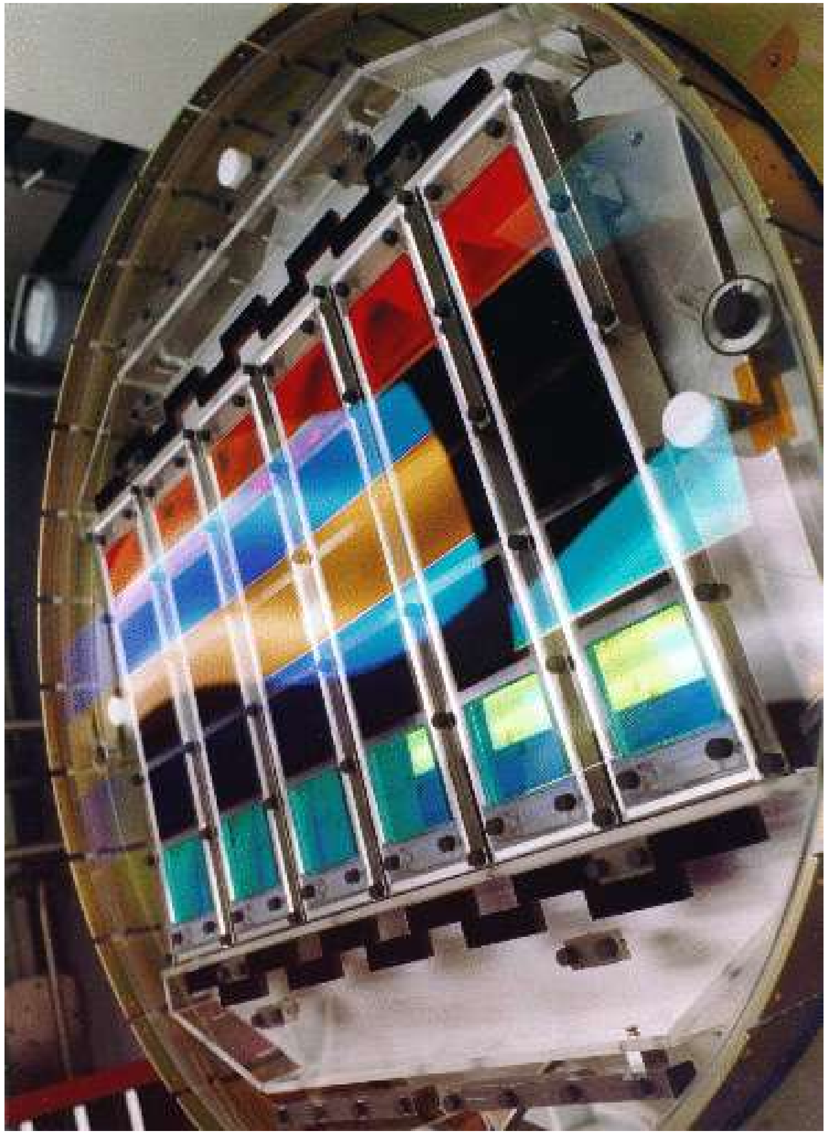
SDSS cannot measure detailed spectra for each galaxy – only measures their brightness in five colour bands.

I have provided you with 8000 galaxies each with 5 photometric colours and 1 spectroscopic redshift.

I have provided a verification set of 1000 galaxies where you can see how well your algorithm is working.

There is then a test set of 1000 galaxies, see if you can derive the redshift from the 5 colours

This will need machine learning.



You could probably just get claude to do all these things for you without you doing anything.

Don’t be completely passive, we have to use machine learning in everyday life but as a scientist you need to understand what is going on.

For example, feel free to ask chatgpt how to set up a neural network (for example), but don’t just give it the data and ask it to solve the problem for you.

Please ask questions to me and to the demonstrators.

Schedule

1230 Lunchtime

1600-1700 we can discuss results together.